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# Love-Type Waves in a Rotating Fibre-Reinforced Functionally Graded Medium with Magnetic Field, Gravity, and Surface Stress Interactions

**Madhumita Kundu, Biswajit Saha, Sakti Pada Barik**

***Abstract: This study investigates the propagation of Love waves in a rotating, fibre-reinforced functionally graded medium (FGM) subjected to the combined effects of a uniform magnetic field, gravity, and surface stress. The magnetic field is oriented to facilitate a two-dimensional analysis of the problem. An analytical expression for the Love wave velocity is derived. The validity of the solution is established by showing that the derived dispersion relation reduces to classical results in the absence of gravity, surface stress, rotation, and fibre reinforcement. Finally, the influence of these governing parameters on the phase velocity is analyzed and illustrated through graphical representations.***

***Keywords: Functionally Graded Material, Love Waves, Surface Stress, Fibre-Reinforced Medium, Magnetic Field, Rotational Effect, Gravity, Wave Number. Subject Classification No.: 74J15, 86A15***

***Nomenclature:***
FRC: Fibre-Reinforced Composite
FGM: Functionally Graded Material

## I. INTRODUCTION

Over the past three decades, the analysis of stress and deformation in fibre-reinforced composite materials has emerged as a pivotal area of research in solid mechanics. The concept of reinforcing brittle matrices with fibrous materials is well-established, with empirical origins in the use of organic fibres such as straw and horsehair in traditional masonry. However, modern materials science has revolutionised these techniques, optimising fibre-matrix interactions to meet the complex performance requirements of advanced engineering applications. The primary objective of reinforcement is to enhance tensile strength and load-bearing capacity without imposing excessive weight penalties. Generally, a fibre-reinforced composite (FRC) comprises three distinct components: (i) the fibres as the dispersed phase, (ii) the matrix as the continuous phase, and (iii) the inter-phase or interface region. Depending on the manufacturing process, fibres may be aligned parallel, distributed randomly, or woven, with the specific arrangement dictating the material's mechanical anisotropy. A. J. Belfield [1] significantly propelled the development of the continuum theory for these materials with pioneering contributions from A. J. M. Spencer, A. C. Pipkin, and T. G. Rogers. The mechanical efficiency of these composites can be further enhanced by incorporating functionally graded characteristics. A Functionally Graded Material (FGM) is a non-homogeneous composite characterized by a gradual, continuous variation in the volume fraction of its constituents, rather than distinct layers. First proposed in 1984 by researchers in Sendai, Japan (Koviani [2], Yamanouchi et al. [3]), FGMs have attracted intensive research interest. Due to their continuously varying macroscopic properties, FGMs often exhibit superior mechanical behavior compared to conventional laminates, particularly in mitigating thermal stress concentrations. Consequently, these materials show great promise in severe operating environments, with applications ranging from aerospace heat shields and thermoelectric generators to automotive braking systems and biomedical implants.

The propagation of mechanical disturbances in solids is a fundamental subject in physics and engineering. The study of wave dynamics in elastic media boasts a distinguished history; early investigations by Poisson, Cauchy, Green, Lamé, and Stokes are comprehensively documented in Love's treatise on the mathematical theory of elasticity. In the late 19th century, the field gained renewed momentum due to its critical applications in geophysics and seismology. It is well established that local excitations, such as earthquakes or underground nuclear explosions, are not detected instantaneously at distant locations; rather, the disturbance propagates over time. A critical aspect of wave motion involves reflection and transmission at boundaries. When a wave encounters an interface separating media with distinct properties, the energy is partitioned into reflected and transmitted components. The Earth, treated as a layered elastic body, supports three primary wave types: (i) Body waves (dilatational and equi-voluminal) propagating through the interior; (ii) Rayleigh waves (1885),

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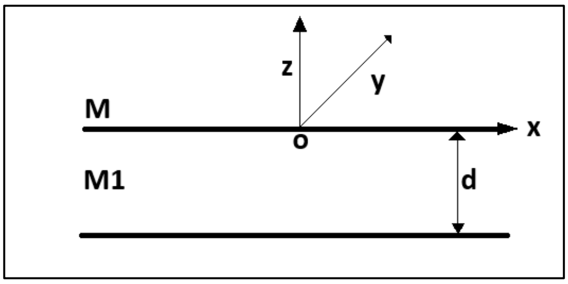
confined to the free surface; and (iii) Love waves (1911), which propagate near the interface of stratified layers. Love waves are of particular interest in seismology due to their destructive potential; unlike body waves, their energy is confined near the surface and dissipates slowly with distance. Mathematically, the existence of Love waves requires that the shear wave velocity in the surface layer be lower than that of the underlying half-space. The analysis of wave propagation in reinforced media is of significant importance in civil engineering and geology. Jassim [4] investigated the inhomogeneous wave equation. Abd-Alla et al. [5] and Das et al. [6] have explored wave dynamics in fibre-reinforced anisotropic elastic media. Furthermore, external fields play a crucial role; early studies by Ranjan [7] and Khan [8] established that gravity and initial stress significantly influence wave velocities. More recently, Maity et al. [9,10], Manna [11], Sahu [12], and Ke [13] investigated propagation in rotating fibre-reinforced poroelastic solids under uniform magnetic and magneto-thermoelastic conditions. Additionally, material inhomogeneity-where physical properties vary with position-has attracted considerable research attention. The anisotropic nature of fibre-reinforced materials, combined with functional gradation, presents a complex yet vital area of study for modern engineering, by Markham [14] and Zorammuana [15]. For instance, Acharya [16] and Islam [17] analysed Love waves under high initial stress in inhomogeneous media.

This paper addresses the propagation behaviour of Love waves in a rotating, fibre-reinforced functionally graded medium under the action of magnetic, gravitational, and surface-stress fields. The governing equations of motion are developed to account for rotation, gravity, surface stress, and the effects of an applied magnetic field. The dependence of Love wave velocity on various parameters is examined and presented graphically.

## II. FORMULATION OF THE PROBLEM

To investigate the propagation of surface waves, we consider a composite structure consisting of two functionally graded, fibre-reinforced elastic media, denoted as $M$ and $M_1$ (Fig. 1) The semi-infinite medium $M$ occupies the region ( $Z \geq 0$ ), while $M_1$ constitutes a finite layer of thickness $d$ defined by $-d \leq Z \leq 0$. The interface between the two media is assumed to be perfectly bonded. A Cartesian coordinate system is established with the origin. $O$ located at the interface and the $Z$-axis directed vertically upward. Furthermore, the entire system is immersed in a uniform magnetic field $H$.




**[Fig.1: Geometry of the Problem]**

### A. Assumptions

- The half-space is under a magnetic field along the $y$-axis, $H_0 = (0, H_0, 0)$.
- The half-space rotates with angular velocity $\Omega = (0,0,\Omega)$.
- Fibres are arranged in parallel along $\alpha = (1,0,0)$.
- Gravity acts along $(0,0,1)$.
- The wave propagates along the $x$-axis, confined near the free surface $z = 0$ and vanishing as $z \to \infty$.

Based on these assumptions, the problem is treated as two-dimensional within the $X - Z$ plane. In the presence of rotation and the applied magnetic field, an induced magnetic field $h = (0, h, 0)$, an electric field $E$, and a current density $J$ are generated. Let $\mu_0$ denote the magnetic permeability; the total magnetic induction is defined as $B = \mu_0 H$, where the total magnetic field intensity is the sum of the primary and induced fields, $H = H_0 + h$. Letting $u = u_i(X, t)$ denote the displacement vector, the governing linearized electrodynamic equations are given by:

$$\nabla \times h = J + \varepsilon_0 \dot{E}, \qquad (1)$$
$$\nabla \times E = -\mu_0 \dot{h}, \qquad (2)$$
$$\nabla \cdot h = 0, \qquad (3)$$
$$E = -\mu_0(\dot{u} \times H) \qquad (4)$$

Where $\nabla$ is Hamilton's operator, $\varepsilon_0$ is the electrical permeability, and $\boldsymbol{u}$ is the dynamic displacement vector. Here, we ignore the small effect of temperature radiation on the current density vector $\boldsymbol{J}$. The deformation is assumed to be small, and the dynamic displacement vector is measured relative to a steady-state reformed position.

Following Belfield [1], the stress-strain relations for a linearly fibre-reinforced elastic medium may be expressed in tensor form as

$$\tau_{ij} = \lambda \epsilon_{kk}\delta_{ij} + 2\mu_T \epsilon_{ij} + \alpha\left(a_k a_m \epsilon_{km}\delta_{ij} + a_i a_j \epsilon_{kk}\right) + 2(\mu_L - \mu_T)\left(a_i a_k \epsilon_{kj} + a_j a_k \epsilon_{ki}\right) + \beta\left(a_k a_m a_i a_j \epsilon_{km}\right),$$

where $\tau_{ij}$ are the Cartesian components of the stress tensor, $\epsilon_{ij}$ are the strain components related to the displacement vector? $u_i . \lambda, \mu_T$ are elastic constants, $\alpha, \beta, (\mu_L - \mu_T)$ are reinforcement parameters, and $\boldsymbol{a} = (a_1, a_2, a_3)$ such that $a_1^2 + a_2^2 + a_3^2 = 1$.

In the absence of body force, the equation of motion for a rotating elastic medium can be written as

$$\tau_{ij,j} + F_i = \rho(\ddot{u}_i + (\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u}))_i + 2(\boldsymbol{\Omega} \times \dot{\boldsymbol{u}})_i), \qquad (6)$$

where $\rho$ denotes the material density. The body force components arising from the applied magnetic field and gravitational effects are given by

$$\boldsymbol{F} = \boldsymbol{J} \times \boldsymbol{B} + \rho \boldsymbol{g}\left(w_{,x}, 0, -u_{,x}\right), \boldsymbol{u} = (u_1, u_2, u_3) = (u, 0, w). \qquad (7)$$

From (1)-(4), and upon neglecting the cross-products of $h$ and $u$ (along with their derivatives), the governing equations for the rotating fibre-reinforced medium, acting under a magnetic field and gravity, may be written as:



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$$\frac{\partial\tau_{11}}{\partial x}+\frac{\partial\tau_{13}}{\partial z}+\mu_0(\boldsymbol{J}\times\boldsymbol{H})_1+\rho g\frac{\partial w}{\partial x}=\rho\left[\ddot{u}-\Omega^2u+2\Omega\frac{\partial w}{\partial t}\right] \quad (8)$$

$$\frac{\partial\tau_{21}}{\partial x}+\frac{\partial\tau_{23}}{\partial z}+\mu_0(\boldsymbol{J}\times\boldsymbol{H})_2=\rho\ddot{v} \quad (9)$$

$$\frac{\partial\tau_{31}}{\partial x}+\frac{\partial\tau_{33}}{\partial z}+\mu_0(\boldsymbol{J}\times\boldsymbol{H})_3-\rho g\frac{\partial u}{\partial x}=\rho[\ddot{w}-\Omega^2w-2\Omega\dot{u}] \quad (10)$$

Both the upper half-space ( $M$ ) and layered medium ( $M_1$ ) are assumed functionally graded. The elastic parameters $(\lambda,\mu_T;\lambda',\mu'_T)$, reinforcement parameters $(\alpha,\beta,\mu_L;\alpha',\beta',\mu'_L)$ and mass density parameter ( $\rho;\ \rho'$ ) are modelled to vary exponentially along the vertical coordinate $z$, such that:

$$\lambda=\hat{\lambda}e^{kz},\alpha=\hat{\alpha}e^{kz},\beta=\hat{\beta}e^{kz},\rho=\hat{\rho}e^{kz},\mu_L=\hat{\mu}_Le^{kz},\mu_T=\hat{\mu}_Te^{kz} \quad (11)$$

for the half-space and for the layered medium, where $k$ and $k'$ are real constants.

$$\lambda'=\hat{\lambda}'e^{k'z},\alpha'=\hat{\alpha}'e^{k'z},\beta'=\hat{\beta}'e^{k'z},\rho'=\hat{\rho}'e^{k'z},\mu'_L=\hat{\mu}'_Le^{k'z},\mu'_T=\hat{\mu}'_Te^{k'z} \quad (12)$$

The reinforcing fibres are assumed to be aligned parallel to the unit vector $\boldsymbol{a}=(1,0,0)$. Consequently, the constitutive relations yield the following relevant components of the stress tensor:

$$\tau_{11}=[(\hat{\lambda}+2\hat{\alpha}+4\hat{\mu}_L-2\hat{\mu}_T+\hat{\beta})\epsilon_{11}+(\hat{\lambda}+\hat{\alpha})(\epsilon_{22}+\epsilon_{33})]e^{kz} \quad (13)$$

$$\tau_{22}=[(\hat{\lambda}+\hat{\alpha})\epsilon_{11}+(\hat{\lambda}+2\hat{\mu}_T)\epsilon_{22}+\hat{\lambda}\epsilon_{33}]e^{kz} \quad (14)$$

$$\tau_{33}=[(\hat{\lambda}+\hat{\alpha})\epsilon_{11}+\hat{\lambda}\epsilon_{22}+(\hat{\lambda}+2\hat{\mu}_T)\epsilon_{33}]e^{kz} \quad (15)$$

$$\tau_{12}=2\hat{\mu}_L\epsilon_{12}e^{kz},\tau_{13}=2\hat{\mu}_L\epsilon_{13}e^{kz},\tau_{23}=2\hat{\mu}_T\epsilon_{23}e^{kz} \quad (16)$$

In the analysis below, for notational convenience, we shall use only parameters and politely remove their hats. Application of (11)-(16) into the equations (8)-(10) and, on simplification, the governing dynamical equations of motion for the medium $M$ reduce to

$$\begin{aligned}(A_{22}+\mu_0H_0^2)w_{,zz}+(B_2+\mu_0H_0^2)u_{,xz}+\mu_Lw_{,xx}-\mu_0H_0h_{,z}-\varepsilon_0\mu_0^2H_0^2\ddot{w}\\ -\rho gu_{,x}+k(\lambda+\alpha)u_{,x}+kA_{22}w_{,z}=\rho(\ddot{w}-\Omega^2w-2\Omega\dot{u})\end{aligned} \quad (17)$$

$$\mu_Lv_{,xx}+\mu_Tv_{,zz}+k\mu_Tv_{,z}=\rho\ddot{v} \quad (18)$$

$$\begin{aligned}(A_{22}+\mu_0H_0^2)w_{,zz}+(B_2+\mu_0H_0^2)u_{,xz}+\mu_Lw_{,xx}-\mu_0H_0h_{,z}-\varepsilon_0\mu_0^2H_0^2\ddot{w}\\ -\rho gu_{,x}+k(\lambda+\alpha)u_{,x}+kA_{22}w_{,z}=\rho(\ddot{w}-\Omega^2w-2\Omega\dot{u})\end{aligned} \quad (19)$$

were

$$A_{11}=\lambda+2\alpha+4\mu_L-2\mu_T+\beta,A_{22}=\lambda+2\mu_T,B_2=\lambda+\alpha+\mu_L$$

From (1) (the magnetic equations), we may write the induced magnetic field component as

$$h=-H_0(u_{,x}+w_{,z}) \quad (20)$$

Substituting equation (20) in the above, the following dynamical equations of motion are valid for the medium $M$ can be expressed as

$$(A_{11}+\mu_0H_0^2)u_{1,xx}+(B_2+\mu_0H_0^2)w_{,xx}+\hat{\mu}u_{1,zz}+\rho gw+\hat{\mu}k(w_z+u_x)=(\rho+\epsilon_0\mu_0^2H_0^2)\ddot{u}-\rho\Omega^2u+2\rho\Omega i\dot{w} \quad (21)$$

$$\mu v_{,xx}+\mu_Tv_{,zz}+k\mu rv_{,z}=\rho\dot{v} \quad (22)$$

$$(A_{22}+\mu_0H_0^2)w_{,zz}+(B_2+\mu_0H_0^2)u_{1,zz}+\mu w_{,xx}+k(\lambda+\alpha)u_z$$

$$+kA_{22}w_z-\mu_0^2H_0^2c_0w=\rho(\ddot{w}-\Omega^2w-2\Omega i\dot{u}+gu_x) \quad (23)$$

Following the same procedure for the layer medium $M_1$, We get the following dynamical equations:

$$\begin{aligned}(A'_{11}+\mu_0H_0^2)u'_{,xx}+(B'_2+\mu_0H_0^2)w'_{,xz}+\mu'_Lu'_{,zz}+\rho'gw'_{,x}+\mu'_Lk(w'_{,x}+u'_{,z})\\ =(\rho'+\varepsilon_0\mu_0^2H_0^2)\ddot{u}'-\rho'\Omega^2u'+2\rho'\Omega\dot{w}'\end{aligned} \quad (24)$$

$$\mu'_Lv'_{,xx}+\mu'_Tv'_{,zz}+k\mu'_Tv'_{,z}=\rho'\ddot{v}' \quad (25)$$

$$(A'_{22} + \mu_0 H_0^2)w'_{,zz} + (B'_2 + \mu_0 H_0^2)u'_{,xz} + \mu'_L w'_{,xx} + k'(\lambda' + \alpha')u'_{,x} + k' A'_{22} w'_{,z}$$
$$-\varepsilon_0 \mu_0^2 H_0^2 \ddot{w}' = \rho'\left(\ddot{w}' - \Omega^2 w' - 2\Omega \ddot{u}' + g u'_{,x}\right) \qquad (26)$$

Where primes (') signify the constraints for medium $M_1$.

## III. BOUNDARY CONDITIONS

Love waves are characterized exclusively by horizontal shear motion; thus, only the transverse displacement $v$ is non-zero. The associated boundary conditions are defined as follows:

$$v = v' \quad \text{at } z = 0 \qquad (27)$$
$$\tau_{23} = \tau'_{23} \quad \text{at } z = 0 \qquad (28)$$
$$\tau'_{23} + \Sigma_{2\alpha,\alpha} - \rho_s \ddot{v}' = 0 \quad \text{at } z = d \qquad (29)$$

where the surface elasticity tensor $\Sigma_{i\alpha}$ follows Gurtin and Murdoch [18] and

$$\Sigma_{i\alpha} = \begin{cases} \delta_{i\alpha}\sigma + (\lambda_s + \sigma)u_{v,v}\delta_{i\alpha} + \mu_s u_{\alpha,i} + (\mu_s - \sigma)u_{i,\alpha}, & i, \alpha = 1,2 \\ \sigma_{i3}, & i = 3 \end{cases} \qquad (30)$$

## IV. SOLUTION OF THE PROBLEM

### A. Half Space

Assuming a solution for the upper half-space given by

$$v(x,z,t) = v_1(z)e^{i\omega(x-ct)} \qquad (31)$$

where $c$ is the phase velocity of simple harmonic waves and $\omega$ is the wave number. Substituting it into Equation (22), we derive:

$$\mu_T v_1''(z) + k\mu_T v_1'(z) - \omega^2(\mu_L - \rho c^2)v_1(z) = 0. \qquad (32)$$

To eliminate the first-order derivative, we employ the transformation $v_1(z) = \varphi(z)v_2(z)$. By setting the coefficient of $v_2'(z)$ to zero, we determine that $\varphi(z) = e^{-kz/2}$. This leads to the reduced equation.

$$v_2''(z) - M^2 v_2(z) = 0 \qquad (33)$$

where the parameter $M$ is defined by

$$M^2 = \frac{k^2}{4} + \frac{\omega^2(\mu_L - \rho c^2)}{\mu_T}$$

To ensure the solution remains bounded as well $z \to \infty$, we reject the exponentially growing component and select

$$v_2(z) = Be^{-Mz}$$

where $B$ is an arbitrary constant. Consequently, the final solution is given by:

$$v(x,z,t) = Be^{-\left(\frac{k}{2}+M\right)z} e^{i\omega(x-ct)} \qquad (34)$$

### B. Layer Medium

For the lower-layered medium, we postulate the upper half-space

$$v'(x,z,t) = v_3(z)e^{i\omega(x-ct)} \qquad (35)$$

We introduce the transformation

$$v_3(z) = e^{-\frac{k'z}{2}} v_4(z) \qquad (36)$$

to remove the first-derivative term, resulting in the canonical form:

$$\frac{\partial^2 v_3}{\partial z^2}(z) + N^2 v_3(z) = 0 \qquad (37)$$

where the parameter $N$ is defined by

$$N^2 = \frac{\omega^2(\rho c^2 - \mu'_L)}{\mu'_T} - \frac{k'^2}{4}$$

The solution for the lower medium will be of the form.

$$v'(x,z,t) = e^{-\frac{k'z}{2}} e^{i\omega(x-ct)}[C\cos(Nz) + D\sin(Nz)] \qquad (38)$$

where $C$ and $D$ are arbitrary constants.

To obtain the complete solution, we must determine the unknowns. $B, C$ and $D$ under the

boundary conditions of the problem. Utilization of boundary conditions (27) and (28) at $z = 0$ yields

$$B = C \qquad (39)$$

and

$$\mu_T\left(M - \frac{k'}{2}\right)B = \mu'_T\left(-\frac{k'}{2}C + DN\right)$$
$$\Rightarrow \mu_T(2M - k')B + k'\mu'_T C - 2N\mu'_T D = 0 \qquad (40)$$

Use of boundary Condition (29) at $z = d$ gives

$$C\left[\cos(Nd)\left(-\frac{k'}{2}\mu'_T - \omega^2\mu_s + \rho_s\omega^2 c^2\right) - N\mu'_T \sin(Nd)\right] + D\left[\sin(Nd)\left(-\frac{k'}{2}\mu'_T - \omega^2\mu_s + \rho_s\omega^2 c^2\right) + N\mu'_T\cos(Nd)\right]$$
$$= 0 \qquad (41)$$

Elimination of the arbitrary constants $B, C, D$ from system of equations (39), (40) and (41) yields the following determinantal equation:

$$\begin{vmatrix} 1 & -1 & 0 \\ \mu_T(2M - k) & k\mu'_T & -2N\mu'_T \\ 0 & Q_1 & Q_2 \end{vmatrix} = 0 \qquad (42)$$

Expansion and simplification of the determinant (42) lead to the dispersion relation:

$$\tan(Nd) = \frac{N\left[\mu_T(k-2M) - k'\mu_T' + 2\mu_T'\left(\frac{k'}{2} + \chi^2\right)\right]}{\mu_T(2M-k) + k'\mu_T' - 2N^2\mu_T'} \quad (43)$$

where the terms $Q_1, Q_2$ and parameter $\chi$ are given by $Q_1 = \left(\frac{k'}{2} - \omega^2\chi^2\right)\cos(Nd) - N\sin(Nd), Q_2 = \left(-\frac{k'}{2} - \omega^2\chi^2\right)\sin(Nd) + N\cos(Nd)$, and $\chi^2 = \frac{\mu_S - \rho_S c^2}{\hat{\mu}_T'}$.

Equation (43) is the characteristic equation (or dispersion relation) governing the wave velocity of Love-type waves propagating in the functionally graded fibre-reinforced model (Figure 1), considering the influence of rotation, gravity, surface stress, and a static initial magnetic field $(0, H_0, 0)$. Significantly, the dispersion relation (43) is independent of the terms associated with the rotation, gravity, surface stress and the initial magnetic field. Consequently, the presence of these external fields does not modulate the Love wave velocity for the specific configuration under study.

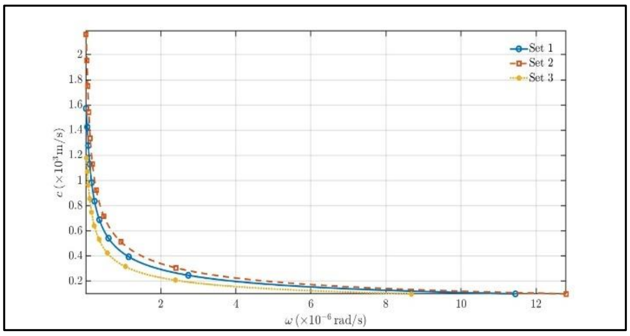
## V. NUMERICAL RESULTS AND DISCUSSIONS

This study investigates the influence of functionally graded parameters, surface stress, magnetic fields, rotation, gravity, and fibre reinforcement on the propagation of Love waves. To numerically evaluate these effects, three distinct media configurations-designated as Fibre-1, Fibre-2, and Fibre-3- were analysed. The material parameters utilized for these models were adopted from the established works of Maity [10], Markham [14], and Zorammuana [15], and are detailed below:

$$\lambda = 5.65 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \mu_T = 2.46 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \mu_L = 5.66 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \rho = 2.26 \times 10^3\mathrm{Kg \cdot m^{-3}}$$
$$\lambda = 7.59 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \mu_T = 3.5 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \mu_L = 7.07 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \rho = 1.6 \times 10^3\mathrm{Kg \cdot m^{-3}}$$
$$\lambda = 9.4 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \mu_T = 1.89 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \mu_L = 2.45 \times 10^9\ \mathrm{N \cdot m^{-2}}, \quad \rho = 1.7 \times 10^3\mathrm{Kg \cdot m^{-3}}$$

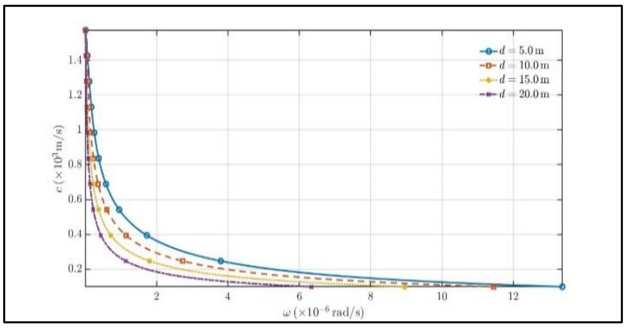
Although the general analysis allows for arbitrary directional vectors, the numerical computations are restricted to specific orientations to facilitate computation and to highlight key trends in wave velocity under fibre-reinforced functionally graded media. Adopting the parameters detailed above, this numerical analysis examines the propagation behaviour of Love waves under varying media conditions.

Figures $2-6$ present the Love wave velocity plotted against the wavenumber for different configurations of reinforcement and functional grading. A notable observation across these figures pertains to the long-wavelength limit (small wavenumber, $\omega \to 0$): the wave velocity approaches its maximum value. Conversely, an increase in wavenumber under any of the examined conditions results in a decrease in wave velocity. It is further observed that, in all cases considered, the wave velocity asymptotically approaches zero as the wavenumber increases. A second significant finding is the influence of applied stress: for a fixed wavenumber, the wave velocity is directly correlated with, and increases with, the applied surface stress.



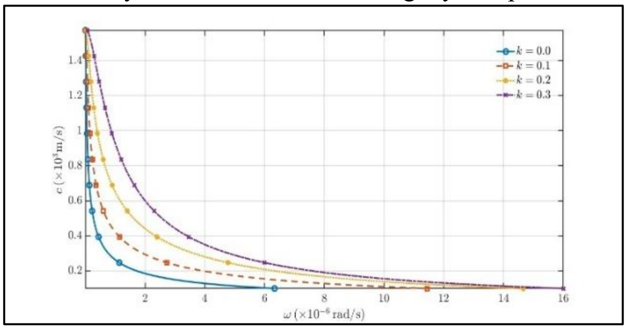

**[Fig.2: Variation of Love Wave Velocity on Different Media]**
**$[(\mu_T - \mu_L) = 3.20 * 10^9$ ( Set -1), $3.57 * 10^9$ ( Set -2), $0.56 * 10^9$ ( Set -3)]**

Figure 2 depicts the variation of Love wave velocity $c$ with wave number $\omega$. It specifically details the impact of the reinforcing parameter on the Love wave velocity across three distinct material sets (Set 1, Set 2, and Set 3). The results clearly indicate that for a constant wave number, the Love wave velocity is inversely proportional to the magnitude of the reinforcing parameter values $(\mu_T - \mu_L)$.




**[Fig.3: Effect of Layer Depth $d$ on Love Wave Velocity]**

The dependency of the Love wave velocity as a function of layer depth (thickness, d) is depicted in Figure 3. For constant values of all other governing parameters in the study, the wave velocity decreases with increasing layer depth.




**[Fig.4: Effect of Functionally Graded Parameter $k$ of a Half-Space on the Love Wave Velocity]**

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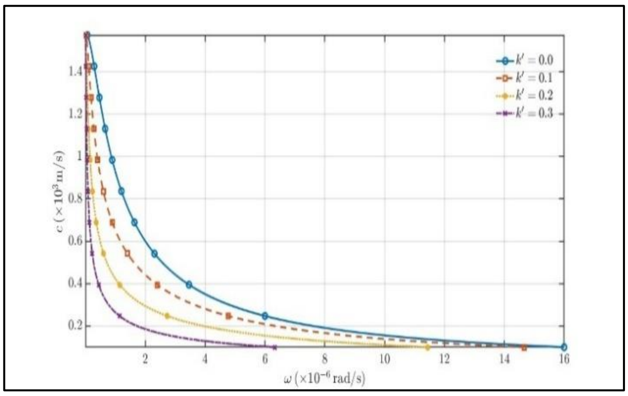
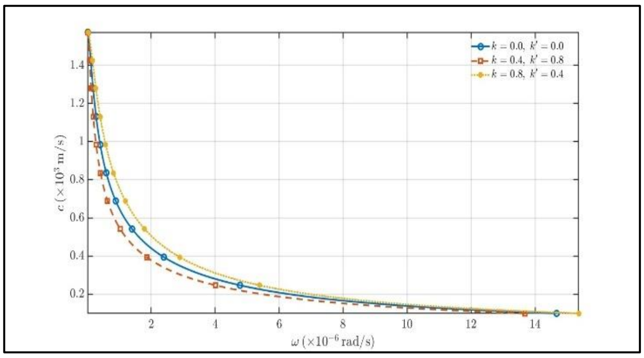



**[Fig.5: Effect of Functionally Graded Parameter $k'$ of the Layer Medium on the Love Wave Velocity]**

Figures 4 and 5 demonstrate the effect of functionally graded characteristics on Love wave velocity. In the upper half-space, the velocity magnitude decreases as the non-homogeneity parameter, $k$, increases for a fixed wave number (Fig. 4). In contrast, the layer medium exhibits the opposite behaviour. As shown in Figure 5, an increase in the non-homogeneity parameter, $k'$It is associated with an increase in Love wave velocity, suggesting that the impact of functional grading on wave speed is medium-dependent.




**[Fig.6: Comparison of Love Wave Velocity of Homogeneous and Non-Homogeneous Medium]**

Finally, Figure 6 illustrates the special case in which both the half-space and the layer are modelled as homogeneous fibre-reinforced media. The blue curve denotes the homogeneous medium, while the red and yellow curves correspond to the non-homogeneous medium. Both the theoretical and numerical analyses demonstrate strong agreement with the results reported by Acharya [16]. Notably, the wave velocity expression in Equation (43) reduces to the corresponding formulation derived by Acharya [16], as shown below:

$$\omega d = \frac{1}{\left\{c_T^2 \times \left(\frac{\mu_L'}{\mu_T'} - \frac{\mu_L'}{\mu_T}\right)\right\}^{1/2}} \tan^{-1}\left[\left\{\frac{\left(\frac{\mu_L}{\mu_T} - c_T^2 \times \frac{\mu_L'}{\mu_T} \cdot \frac{\rho'}{\rho}\right)}{c_T^2 \times \left(\frac{\mu_L'}{\mu_T'} - \frac{\mu_L'}{\mu_T}\right)}\right\}^{1/2}\right]$$

in which $c_T = c/\sqrt{\mu_L'/\rho'}$.

## VI. CONCLUSION

The presence of fibre reinforcement and non-homogeneity significantly influences Love wave velocity. Key observations include:

(i) For a fixed wave number, the wave velocity varies proportionally with the magnitude of the reinforcement parameter $(\mu_T - \mu_L)$.

(ii) The wave velocity exhibits a direct dependence on the functionally graded parameter $k$ of the half-space, whereas it varies inversely with the functionally graded parameter $k'$ of the layer. Significantly, the Love wave velocity is independent of terms associated with rotation, gravity, surface stress, and the initial magnetic field. Consequently, these external fields do not modulate the wave velocity within the specific configuration under study.

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## DECLARATION STATEMENT

Some of the references cited are outdated, noted explicitly as [1], [2], [3], [4], [5], [7], [8], [11], [12], [13], [14], [15], [16], and [18]. However, these works remain significant for the current study, as they are pioneering in their fields.

As the article's author, I must verify the accuracy of the following information after aggregating input from all authors.

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- **Author's Contributions:** The authorship of this article is contributed equally to all participating individuals.

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- **Two scholars** have been successfully awarded their PhD degrees.
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