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IIR Multiple Notch Filter Design for Power Line Interference Removal

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Abstract— Digital IIR notch filter has been employed in various practical applications i.e in communication systems, medical science and many more to eliminate unwanted narrowband interference with known frequency. It is difficult to filter noise from these signals, and errors resulting from filtering can distort a biomedical signal. This paper presents the design technique and implementation of IIR multiple notch filter by the application of suitable pole placement technique. This technique gives the fixed value of parameters used to design of multiple IIR notch filter for removal this harmonically distributed kind of interference.

Index Terms—Digital IIR multiple notch filters, mathematical modeling, Pole Re-position.

I. INTRODUCTION

Power line interference comes in many of the system during their operation because of the presence of power line that mainly uses 50Hz in India and 60Hz in USA. The interference caused due to these lines mainly affects the three application of science one is in medical science named ECG and other in telecommunication as hum in audio system and during speech recording. Generally, the frequency band of the ECG signal is 0.05 to 100Hz, and the ECG signal includes 60Hz power line noise, baseline wander due to respiration, and muscle artifacts resulting from the movement of electrodes during measurement. 60Hz power line noise can affect the Q- and P-waves of the ECG signal (Fig.1), generating errors during the diagnosis of arrhythmia or myocardial infarction. Power line noise can cause errors by distorting the ECG signal during the measurement of the QRS complex interval or the QT interval, which are important parameters in diagnosis. Hum is an unwanted 60 Hz tone or 50 Hz in Europe maybe with harmonics. If the harmonics are especially strong, the hum becomes an edgy buzz. Your sound system also might be plagued by RFI (Radio Frequency Interference). It's heard as buzzing, clicks, radio programs, or "hash" in the audio signal. RFI is caused by CB transmitters, computers, lightning, radar, radio and TV transmitters, industrial machines, cell phones, auto ignitions, stage lighting, and other sources. In order to remove 60Hz power line noise and maybe it's harmonics a notch filter is employed that removes a particular frequency from a signal and has a frequency response that falls to zero over a narrow range of frequencies (i.e. a 60 Hz notch may block signals from 59.5 – 60.5 Hz). Notch filter actually can also be perceived as a band stop filter with a high Q factor, i.e. it often wants to filter out the undesired signal in the specific frequency (e.g. noise) only. However, the conventional band stop filter usually has a relatively wide stop band.

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There also are many applications of notch filters in the field of signal processing such as removing the power line interferences in electrocardiograms (ECG)[1], cutting noise in broadcast TV, extending the spectrum analyzer range, rejecting the interference in ultra wideband (UWB)[2] radio system etc.

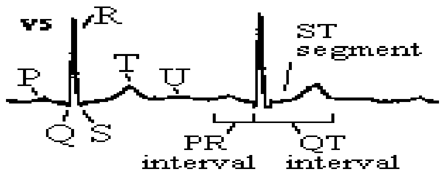

Figure1: ECG Waveform Trace

The paper gives the idea about the fixed notch filter with respect to its parameter values that are used in the designing of IIR notch filter. After the complex calculation and iteration for notch filter these design parameters are decided to get better result at particular notch.

II. BACKGROUND

To design a digital notch filter, there are many methods for IIR and FIR filter design. The major measures to design an IIR digital notch filter are (1) Analog filter transformation [3] (2) All pass filter implementation [4,5] (3) Pole-zero placement technique[6].The multiple notch filters can be used for the removal of multiple narrowband or multiple frequency interferences such as the removal of single-tone interference and its harmonics.

FIR (finite impulse response) notch filter cannot achieve a desired sharpness of response (means high Q value) with many fewer coefficients, the IIR notch filter can implement unusual characteristics such as being an all pass. Secondly, having less number of side lobes in the stop band and implementation required few parameters, less memory requirements and low computational complexity.

The transfer function of the IIR single notch filter is based on that of the all-pass filter, except that the gain at notch frequency is exactly zero. The following subsections will explain each of the design techniques and their revolution.

A. Previous IIR Single Notch Filter Design Technique

The frequency response of IIR single notch filter can be described as shown in (1) while the transfer function [8]of the IIR single notch filter can be described as shown in (2),

$$H(e^{j\omega_o} = \begin{cases} 0, & \omega_o \\ 1, & \text{otherwise} \end{cases} \qquad (1)$$

$$H(z) = \frac{b_0(1-2cos\,\omega_0 z^{-1}+z^{-2})}{(1-2rcos\,\omega_0 z^{-1}+r^2 z^{-2})} \qquad (2)$$

Where:

H(z)= Is the transfer function of IIR single notch filter.

ω_0 = Is the pole-zero angle in z-plane.

r = Is the distance between the pole and the origin.

b_o = Is the constant Coefficient of IIR notch filter.

From Eq.(2), let $\omega_0 = 0.8\pi$ rad/sec and r = 0.6, 0.7, 0.8, 0.9, 0.99. Then, Fig. 1 shows the angle of poles and zeros at the same positions and the magnitude response is shown in Fig.2. According to the parameters above, the results show the asymmetric uncontrollable pass band gains. The trade-off for larger value of *r* is that 1) the frequency response is closer to the ideal case and 2) the stability margin (defined as shortest distance of the largest pole to the unit circle) is narrower.

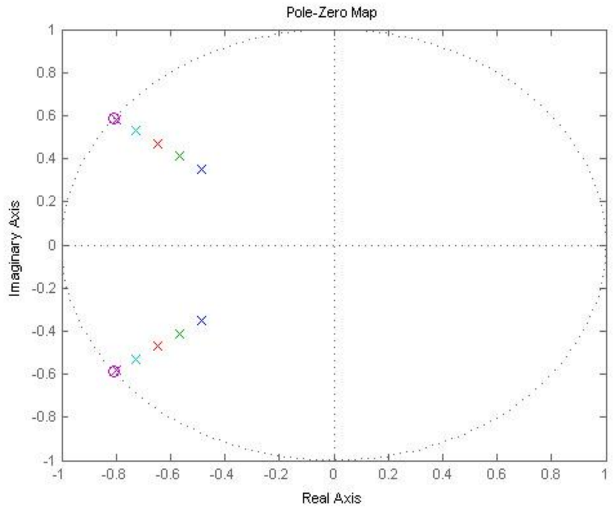

Figure 2: Pole-Zero plot of IIR notch filters before changing the positions of poles.

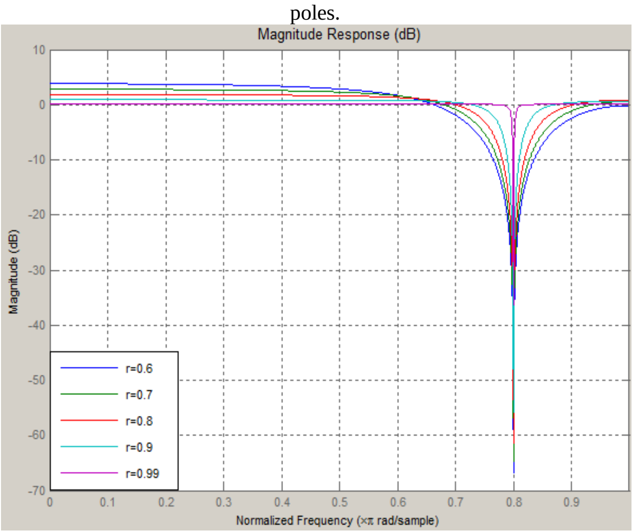

Figure 3: Magnitude responses of the conventional IIR notch filters before applying the pole re-position technique.

Therefore, it is necessary to change the pole positions to the more appropriated positions while calculating the coefficients to control the gain as shown in (3) & (4). The magnitude responses after the modification of pole positions will become as shown in Fig.3. Hence the transfer function of this technique [6] can be expressed as

$$H_n(z) = \frac{b_n(1-2\cos\omega_n z^{-1}+z^{-2})}{(1-2r\cos\varpi_n z^{-1}+r^2 z^{-2})} \qquad (3)$$

Where ϖ_0 represents the adjusted notch frequency which can be represented as

$$\varpi_0 = \cos^{-1}\left[\frac{1+r^2}{2r}\cos\omega_0\right] \qquad (4)$$

The pole-zero plot of this approach is shown in Fig.3 . While the magnitude responses after adjusting the positions of the poles are shown in Fig. 4.

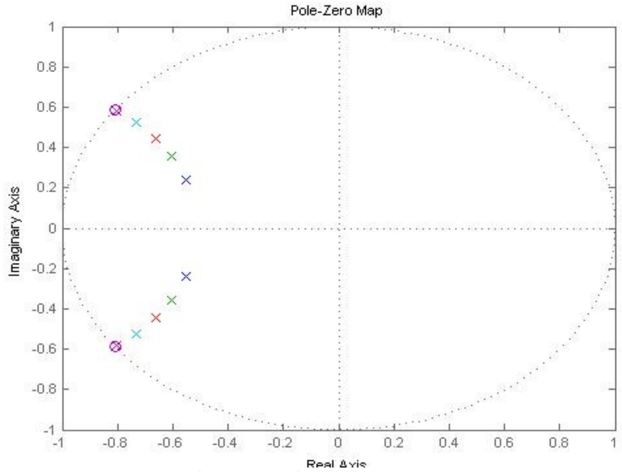

Figure 4: Pole-zero plot of IIR notch filter after changing the pole position.

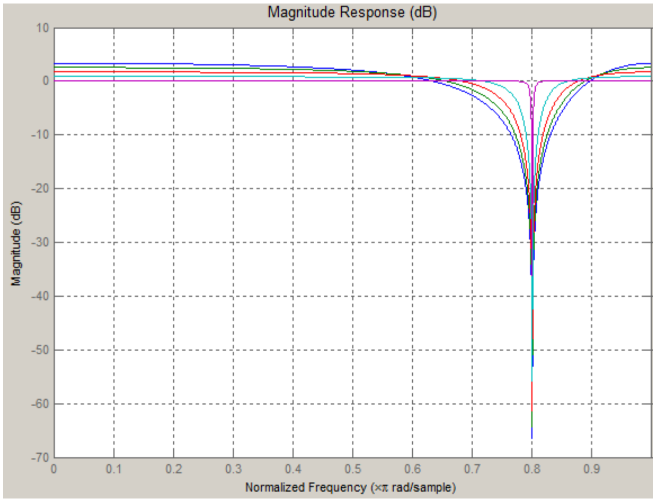

Fig 5: Magnitude responses of the IIR notch filter after applying pole re-position technique.

B. *Effect of Pole Re-position:*

After the pole placement the changes comes at 1) the angle between pole-zero is not same for all pole radius i.e. angle is going to be increased as we move farther away from origin value, 2)And, the asymmetric uncontrollable pass band gains become symmetric after pole re-position.

III. MATHEMATICAL MODELING:

After pole re-position the pass band gain become symmetric but still not equal to unity. So, to make the gain unity at both DC and π frequencies do the mathematical calculations [9] that follow the table 1 as shown,

Table1: Controlled gain for pole re-position technique with N=2

Frequency	DC	ω_1	ω_2	Π
Notch1	k1	0	-	k1/a1
Notch2	k2	-	0	a1.k2
Multiple	k1.k2	0	0	k1.k2

So, calculation of gain at DC frequency by putting $Z=e^{j0}$ in eq.(3) solved as,

$$H_1(e^{j0}) = \frac{b_1(1-2\cos\omega_1(e^{j0})^{-1}+(e^{j0})^{-2})}{(1-2r\cos\varpi_1(e^{j0})^{-1}+r_1^2(e^{j0})^{-2})}$$

i.e. $$k_1 = \frac{b1(2-2cos\,\omega_1)}{(1-2r_1 cos\,\varpi_1 + r_1^2)} \tag{5}$$

Similarly gain at π frequency by putting $Z=e^{j\pi}$ (i.e = -1) in eq.(3) solved as,

$$H_1(e^{j0}) = \frac{b_1(1-2cos\omega_1(e^{j\pi})^{-1} + (e^{j\pi})^{-2})}{(1-2r_1 cos\varpi_1(e^{j\pi})^{-1} + r_1^2(e^{j\pi})^{-2})}$$

i.e $$k_1/a_1 = \frac{b1(2+2cos\,\omega_1)}{(1+2r_1 cos\,\varpi_1 + r_1^2)} \tag{6}$$

From eq.(5)&(6) the transfer function can be rearranged to calculate for $cos\varpi_1$ solved as,

$$k_1 = \frac{b1(2-2cos\omega_1)}{(1-2r_1 cos\varpi_1 + r_1^2)} = a_1 \frac{b1(2+2cos\omega_1)}{(1+2r_1 cos\varpi_1 + r_1^2)}$$

$$cos\varpi_1 = \frac{(1+r_1^2)(a_1 - 1 + cos\,\varpi_1 + a_1 cos\,\varpi_1)}{2r_1(a_1 + 1 - cos\,\varpi_1 + a_1 cos\,\varpi_1)} \tag{7}$$

By defining

$$m_1 = \frac{(a_1 - 1 + cos\varpi_1 + a_1 cos\varpi_1)}{(a_1 + 1 - cos\varpi_1 + a_1 cos\varpi_1)}$$

The value of $cos\varpi_1$ can be written as,

$$cos\varpi_1 = \frac{(1+r_1^2)}{2r_1} m_1 \tag{8}$$

In last to calculate the constant b_1 by substituting $cos\varpi_1$ from eq.(8) to eq.(5) i.e.

$$b_1 = \frac{k_1(1-2r_1 cos\,\varpi_1 + r_1^2)}{(2-2cos\,\omega_1)} \tag{9}$$

The constant filter co-efficient b_n is used in calculation of IIR notch filter transfer function numerator part eq.(9) and the denominator part calculated with the help of modified pole position eq.(4).

Same procedure is repeated for the calculation of second IIR notch filter with reference to table 1 parameters will be taken and in that the two important parameters will be calculated in the form as shown in eq.(10)&(11).

$$m2 = \frac{-a1 + 1 + cos\,\varpi_2 + a1 cos\,\varpi_2}{a1 + 1 + cos\,\varpi_2 - a1 cos\,\varpi_2} \tag{10}$$

In same way the filter coefficient b_2 is given as,

$$b2 = \frac{k2(1-2r2 cos\,\varpi_1 + r_2^2)}{(2-2cos\,\omega_1)} \tag{11}$$

IV. SIMULATION RESULTS

Design the multiple notch filter for power line interference and it's harmonics removal that has application in i.e. one used in medical science and another in telecommunication to remove hum[7] that occur at 60Hz and it's harmonics.

For a multiple notch filter with n=5, the frequency response is given in eq.(12)

$$H(e^{j\omega_o}) = \begin{cases} 0, & \omega_o = 0.15\pi, 0.30\pi, 0.45\pi, 0.60\pi, 0.75\pi \\ 1, & \text{otherwise} \end{cases} \tag{12}$$

To get the transfer function shown in eq.(3), we have to calculate the modified pole re-position parameter $cos\varpi_1$ and for that we first find out the constant m_1 i.e. $m_1 = 0.8900$ and second contant b_1 =0.9896 with the help of these constants we calculate parameter $cos\varpi_1$ and get the transfer function as shown by eq.(13)

$$H_1(z) = \frac{0.9896 - 1.763z^{-1} + 0.9896z^{-2}}{1 - 1.745z^{-1} + 0.9604z^{-2}} \tag{13}$$

The pole-zero plot and magnitude response graph shown as,

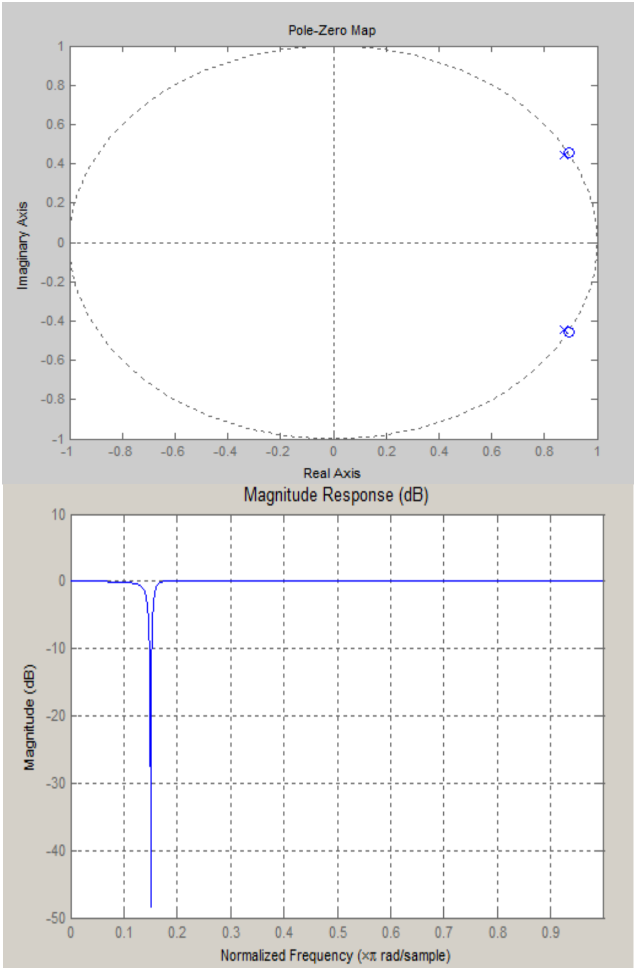

Figure 6: Pole-zero plot and magnitude response of power line notch filter (at 60 Hz)

Now for second notch for power line at 120Hz(2nd harmonic) again we have to calculate the modified pole re-position parameter $cos\varpi_2$ and for that we first find out the constant m_2 i.e. $m_2 = 0.5845$ and second contant b_2 =0.9880 with the help of these constants we calculate parameter $cos\varpi_2$ and get the transfer function as shown by eq.(14)

$$H_2(z) = \frac{0.9880 - 1.162z^{-1} + 0.9880z^{-2}}{1 - 1.146z^{-1} + 0.9604z^{-2}} \tag{14}$$

In the same way we get the transfer funcion for all rest harmonics up to 5th harmonic that is at 300Hz shown by equations (15),(16)&(17).

$$H_3(z) = \frac{0.9859 - 0.3085z^{-1} + 0.9859z^{-2}}{1 - 0.2971z^{-1} + 0.9604z^{-2}} \tag{15}$$

$$H_4(z) = \frac{0.9836 + 0.6079z^{-1} + 0.9836z^{-2}}{1 + 0.6147z^{-1} + 0.9604z^{-2}} \tag{16}$$

$$H_5(z) = \frac{0.9816 - 1.388z^{-1} + 0.9816z^{-2}}{1 + 1.391z^{-1} + 0.9604z^{-2}} \tag{17}$$

Now cascading of these transfer function from eq.(13) to (17) will give the final transfer function of multiple IIR notch filter i.e. described by eq.(18),

$$H_6(z)=\frac{0.8796-1.089z^{-1}+1.977z^{-2}-2.049z^{-3}+}{1-1.246z^{-1}+2.195z^{-2}-2.256z^{-3}+}$$

$$\frac{+2.684z^{-4}-2.423z^{-5}+2.684z^{-6}-2.049z^{-7}+}{+2.874z^{-4}-2.565z^{-5}+2.76z^{-6}-2.081z^{-7}+}$$

$$\frac{+1.977z^{-8}-1.089z^{-9}+0.8796z^{-10}}{+1.945z^{-8}-1.06z^{-9}+0.8171z^{-10}} \tag{18}$$

The pole-zero plot and magnitude response graph shown as,

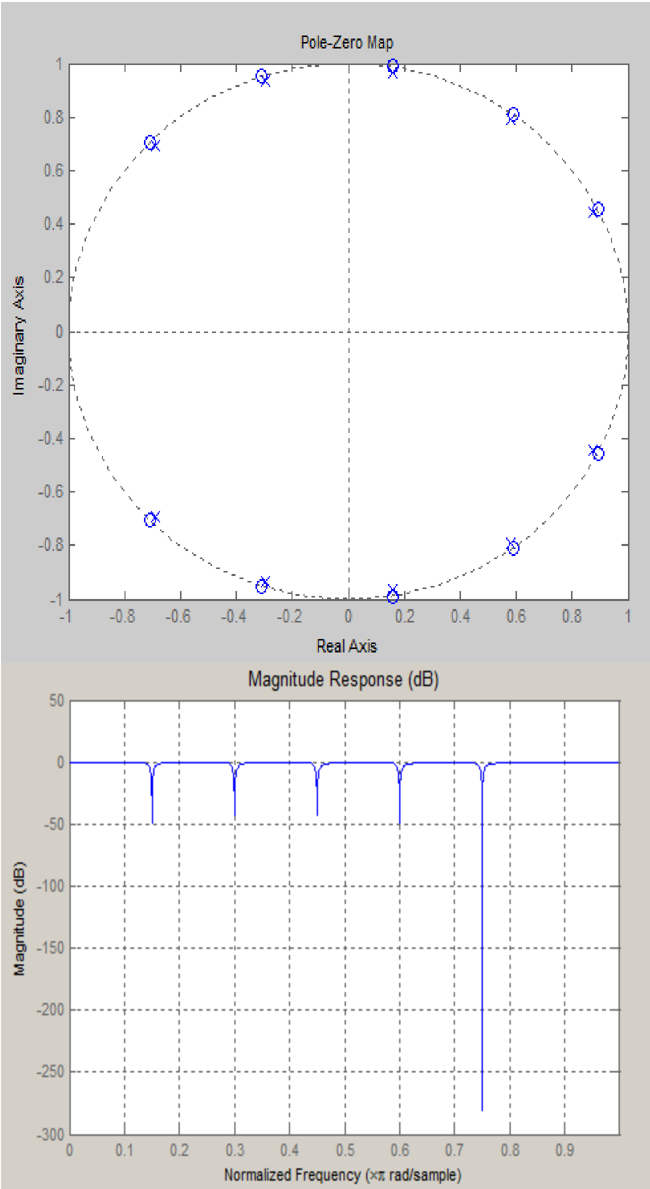

Figure 8: Pole-zero plot and magnitude response of multiple power line interference notches filter

V. CONCLUSION:

The proposed design technique that uses the modified pole re-position technique given the better result for removal of power line interference with harmonic removal. The final graph after cascading all the five transfer function given the overall transfer function shown by eq.(18). The pole-zero plot and magnitude response plot shown by Fig.(7) shows that there is symmetry in pole-zero across the imaginary axis and magnitude response shows all the five notches with a notch bandwidth of nearly 4-5Hz.The two factors used in design of multiple notch filter i.e. constant 'a' and pole radius 'r' are main factor. The bandwidth of the filter is inversely proportional to pole radius i.e.$BW \propto \frac{1}{r_n}$ due to that reason to make sharp notch we choose the pole radius nearly equal to unity. The flatness of magnitude response of this filter is also inversely proportional to that of constant 'a' i.e. $flatness \propto \frac{1}{a}$ due to that reason we take this as much value. So, to have a sharp notch at the required frequencies for multiple frequency notch filter design we choose the two values i.e. pole radius 'r' and constant 'a' to be 0.98 and 0.99 respectively. After choosing these values of pole radius and constant with sampling frequency with a multiplication factor of 2.66 so that it avoids aliasing effect in both the designs we get sharp notches at the resp. frequencies. And, we can use these notch filters to remove the ECG power line interference and hum in audio system and speech recording that also comes at same frequency range with harmonics.

VI. FUTURE SCOPE

As we design this multiple notch filter for two practical applications i.e. ECG and hum in audio system and speech recording up to five notches. We further can extend this project to notch up to ten harmonic frequencies and also for different applications that are helpful for practical purpose e.g. cutting noise in broadcast TV, rejecting the interference in ultra wide band (UWB) radio system that interfered by IEEE 802.11 WLAN services. For the future research direction, since the complexity of IIR multiple notch filter search is not analyzed, the proposed future techniques may improved further this research if the better search algorithm can be considered to get these parameters. And, error that comes along the notch in magnitude plot of few db can be further reduced and bandwidth of notch filter can be reduced much less by using other algorithm in design because that value contradicts with the stability of the filter in future.

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