Design & Implementation of State Estimation Based Optimal Controller Model Using MATLAB/SIMULINK

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Abstract— This paper proposes model based technique to control the horizontal position of a helicopter. The main constraint in the controller design is that not many states are measurable, and that the available sensor information is highly corrupted by noise. Here, the change in the declining angle of rotor is used to steer the helicopter in a straight line. The design is constrained to keep other parameters within the specified limits. The controller design is based on a combination of Kalman filter observer along with optimal linear quadratic Gaussian (LQG) controller. The design is implemented in two steps. First, Kalman filter is used to design an observer that estimates two desired states of a helicopter: rotator angle and horizontal position. Second, state feedback controller gain is estimated using the linear quadratic criterion function. The state controller enhances the regulation performance, while minimizing cost of control effort. Simulation results prove the credibility of Kalman filter observer by comparing the estimated states such as position and angle with the model output. In addition, the performance of LQG controller is examined by incorporating servo control mode that reduces the effort to compute error between reference and measured position.

Index Terms- helicopter system, Kalman filter observer, linear quadratic gaussian controller, state space model

I. INTRODUCTION

The history of control theory can be divided in three periods. These are 'primitive period' that lasted until 1940, 'the classical period', lasted nearly 20 years, and 'the modern period' started from early '60s. The Nyquist and Bode of '30s and the Wiener filter of '40s were applicable to the linear systems only. The primary objective at the time was to express results in terms of transfer functions or impulse responses.

The systems under considerations were quite simple and could easily be modeled by single order differential equations. Further, most of the applications were essentially based on mechanical systems, as the electrical devices available were limited to magnetic coils, and resistors only.

The introduction of state-space model in early 50's brought a new dimension, which helped in analyzing systems that were approximated by higher order differential equations. The Kalman filter in early '60s used state-space approach to approximate a non-linear system of much higher order. In addition, it provided an optimum solution for systems with Gaussian random noise.

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Model based control strategies are quite popular in modern control theories, and applications. This paper models the motion of a helicopter. The helicopter motion is inherently non-linear, but this can be approximately by using three degrees-of-freedom. In literature, several solutions have been proposed for designing a controller. These mainly senses the helicopter position using sensors, and then issues appropriate control signals to steer the helicopter in an appropriate direction. There are several challenges associated with the dynamic nature of helicopter motion as discussed in [1]. They make the designing of an optimal controller a non-trivial task. The main constraints are the limited number of states that are measurable through various sensors and external disturbances like wind gust and model uncertainties as mentioned in [2].

This paper proposes a Linear Quadratic Gaussian (LQG) controller that uses Kalman filtering observer for the prediction of future states such as horizontal position and declining angle. The application of Kalman filter observer in making the corrections recursive by minimizing the error of estimated values for multivariable ship motion control is presented in [2]. Similarly, the extended Kalman filter to estimate the state of heating process and flexible joint manipulator for time varying disturbance is proposed in [3]-[4]. In [5]-[6] the extended Kalman filter and fuzzy Kalman filter has been proposed to estimated the state of positioning system. The implementation of low cost non linear observer for estimating the altitude of a flying object is been proposed in [7]. In [8] adaptive Kalman filter algorithm is been proposed to estimate the state of charge (SOC) of a lithium-ion battery. In literature several observes has been proposed in [9]-[10] based upon Kalman filter estimation techniques. Among these, two types of observer can generally be categorized in 'predictive observer' and 'filtering observer'. In predictive observer approach there is a time delay of one sample between the measurement and estimated state vector, whilst for the filtering observer there is no time delay. In this paper, the observer is designed using the 'filtering observer' approach. The state space (SS) model of a helicopter system has been developed containing two states, the horizontal position and the 'rotor angle' at time t_0 . The advantage of using state space is that a system of much higher order can be approximated by a first order equation.

Furthermore, several solutions proposed in literature to design a LQG controller. A control design based on Linear Quadratic Gaussian / loop transfer recovery (LQG/LTR) is proposed in [11]. The flight dynamic structural model of sub-mini helicopter controller design is given in [12]. A

Lyapunov stability criterion is used to approximate the desired path in the presence of wind disturbance in [13]. A flight inversion approach has been designed for realization of automatic control in [14]. A Linear Quadric Gaussian (LQG)-obstacles based controller design has been proposed in the robots [15]. A state-space model for a helicopter is developed using the uncertainty model of the system [16]. An interacting multiple model (IMM) Kalman filter approach is proposed in [17]. A linear parameter varying (LPV) bases controller design is proposed in [18]. A state space linearized controller design using Euler-Lagrange equations is proposed in [19].

The outline of the paper is as follows. After a brief introduction in this section, the next section gives Mathematical model of a helicopter. This is followed by Kalman filter design observer in Section III. Section IV gives mathematical background on LQG controller design. The results are given in Section V, followed by conclusions in Section VI.

II. MATHEMATICAL MODEL

A state-space model has been derived to understand the dynamic behaviour of a helicopter system by using conceptual model as shown in Figure 1. It has been assumed that the helicopter is moving only in horizontal direction, where horizontal position "$x(t)$" is controlled by changing the decline of the rotor indicated by “$\delta(t)$”. The decline of the helicopter is indicated by angle “$\theta(t)$”.

The model of helicopter system, that describes the movement in horizontal direction, is modeled by two second order equations. The following equation models the decline of helicopter:

$$\ddot{\theta} = \tau_1 \dot{\theta} - \alpha_1 \dot{x} + \vartheta_1 \delta \quad (1)$$

While, second equation describes the helicopter position in horizontal direction,

$$\ddot{x} = g\theta - \alpha_2 \dot{\theta} - \tau_2 \dot{x} + g\delta \quad (2)$$

Where the constants are given as:

$$\tau_1 = 0.415, \quad \alpha_1 = 0.0111, \quad \vartheta_1 = 6.27$$
$$\tau_2 = 0.019, \quad \alpha_2 = 0.0111, \quad g = 9.81$$

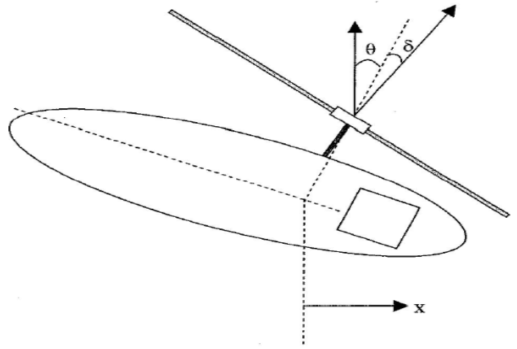

Fig 1. Conceptual model for the helicopter system

A. State Space Modeling in Continuous Time

The state space representation of two second order differential equations as describes in Eq (1) and Eq (2) can be written as:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \quad (3)$$

Where the matrices *A*, *B* and *C* are parameters of the state space model and variable x is the corresponding state variables of helicopter system states, described as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \quad (4)$$

Where, state x_1 representing decline angle (θ) and state x_3 represents horizontal position (x) of helicopter system. Furthermore, input and output state vectors are defined as:

$u = \delta$, and $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta \\ x \end{bmatrix}$

Where, input vector δ, is the decline of the rotor and output vector y_1, y_2 are again the decline angle of the helicopter and horizontal position respectively. Therefore, general state space model of a helicopter system can be derived here:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\tau_1 & 0 & -\alpha_1 \\ 0 & 0 & 0 & 1 \\ g & -\alpha_2 & 0 & -\tau_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ v_1 \\ 0 \\ g \end{bmatrix} \delta \quad (5)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta \\ x \end{bmatrix} \quad (6)$$

Now, substituting numerical constant values in Eq (5) and Eq (6), the general state space model can be written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.415 & 0 & -0.0111 \\ 0 & 0 & 0 & 1 \\ 9.81 & -1.430 & 0 & -0.0198 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 6.27 \\ 0 \\ 9.81 \end{bmatrix} \delta \quad (7)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (8)$$

B. State Space Modeling in Discrete Time

Furthermore, Transforming the model into a discrete state space form, by using Matlab, where the sample interval is chosen as h=0.4 seconds. It is also important to add process and measurement disturbances to model the noise figure. Therefore, general form of discrete state space model can be written as:

$$x(k+1) = Fx(k) + Gu(k) + v(k)$$
$$y(k) = Cx(k) + Du(k) + w(k) \quad (9)$$

Where $v(k)$ and $w(k)$ are the process and measurement noise. Finally, discrete state space model for the helicopter system can be written as:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.9989 & 0.368 & 0 & -0.0008 \\ -0.008 & 0.847 & 0 & -0.0041 \\ 0.7828 & -0.007 & 1 & 0.3985 \\ 3.9090 & 0.216 & 0 & 0.9922 \end{bmatrix} * \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0.4 \\ 2.3 \\ 0.7 \\ 3.8 \end{bmatrix} u(k) + v(k) \quad (10)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + w(k) \quad (11)$$

III. KALMAN FILTER OBSERVER DESIGN

The fundamental objective of designing the Kalman filtering observer is to estimate; state variables of a helicopter system, including decline angle and horizontal position. These states can be used further to design LQG controller to control the helicopter at desired position.

The output from observer is an estimate of state variables such as decline angle and horizontal position, consisting of both predictive and correction part. The predictive part is purely based on the process model and correction part is based on a weighted difference between an actual measurement and predicted measurement [20]-[21]. The Kalman filtering based observer equation can be derived as:

Estimated state = Predicted state + K*correction term

$$\hat{x}(k) = \bar{x}(k) + K\{y(k) - C\bar{x}(k)\} \quad (12)$$

Where, $\bar{x}(k)$ is the prediction vector of state variables i.e. decline angle and position at sample k, the prediction vector can be written as:

$$\bar{x}(k+1) = F\hat{x}(k) + Gu(k) \quad (13)$$

The quality of filtering observer can be examined under the residual analysis i.e. difference between the measured process output and estimated output.

$$e(k+1) = x(k+1) - \hat{x}(k+1) \quad (14)$$

It also determines the degree of correlation between the actual measurement and estimated measurement, if residual becomes zero that proved to be an ideal observer. In Eq (12) the weighted factor K plays a vital role in determining the effect of correcting term for estimating helicopter states that could minimize the error. In general we can make the weighting factor K larger, if the trust in the output $Y(k)$ is high enough or vice versa. To obtain a Kalman gain matrix, Kalman estimator design for discrete-time system is implemented in MATLAB using DLQE (discrete-linear-quadratic-estimator) command [22]-[23], which is given below as:

$$[K, P] = dlqe(F, H, C, R1, R2) \quad (15)$$

It has been assumed that the normally distributed white noise $v(k)$ and $w(k)$ has been added to process states and output, so the covariance matrices for the disturbance $v(k)$ and $w(k)$ are R_v and R_w respectively.

$$R_v = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix}, R_w = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \quad (16)$$

The computed Kalman gain matrix is given below:

$$K = \begin{bmatrix} 0.6736 & 0.0500 \\ 0.3132 & -0.0052 \\ 0.0500 & 0.8489 \\ 0.8913 & 1.3976 \end{bmatrix} \quad (17)$$

The proposed Kalman filter observer equation can be written as:

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \\ \hat{x}_3(k+1) \\ \hat{x}_4(k+1) \end{bmatrix} = \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \\ \bar{x}_4(k) \end{bmatrix} + \begin{bmatrix} 0.6736 & 0.0500 \\ 0.3132 & -0.0052 \\ 0.0500 & 0.8489 \\ 0.8913 & 1.3976 \end{bmatrix} * \left\{ \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \\ \bar{x}_4(k) \end{bmatrix} \right\} \quad (18)$$

Simulink model of the designed Kalman filter observer is shown in Fig.2. In order to keep the simulink model simpler, vector notation for both states and output has been used. In addition, Matrix Gain block has been used to multiply F, G and C matrices and vectors that also provided an opportunity to configure the suitable block. To perform the vector computation wide non-scalar line has been formed.

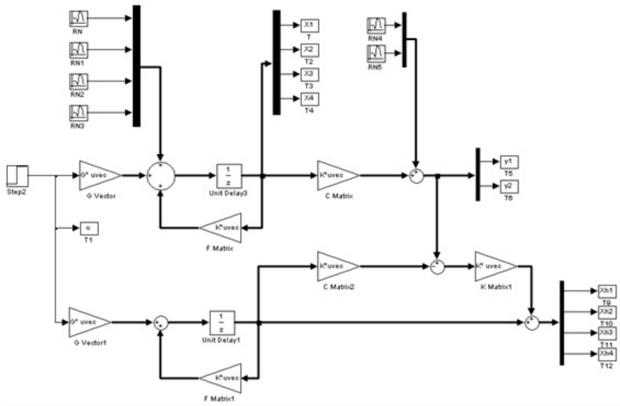

Fig-2: Simulink model of Kalman filter observer

IV. LQG CONTROLLER DESIGN

The focus of this section is to design an optimal controller to control the horizontal position of helicopter at desired state. The purpose of designing a model based optimal controller is to enhance the regulation performance, while minimizing the cost of control effort as well as reducing the disturbance effect. To address this problem, LQG controller can be implemented in two steps: (i) designing of a Kalman filter observer to estimate the desired states that is needed to be control (ii) calculation of state feedback controller gain to minimize the cost function based on linear quadratic criterion function. From the separation principle, the observer together with a state space feedback can be separated into two separate problems. The LQG controller can be formulated as:

$$u(k) = -L(k)\hat{x}(k) \qquad (19)$$

Where, $u(k)$ represent decline of the rotor indicated by δ, $L(k)$ denote the feedback controller gain and $\hat{x}(k)$ represents estimated states of both decline angle and horizontal position from the observer. Furthermore, unlike the pole placement design controller, LGQ controller can be made time varying by incorporating servo control time varying reference. It will allow the helicopter system states to be compared with desired or reference states $x_{ref}(k)$, which will help to move the horizontal position of helicopter to desire level of state. The servo control problem is achieved by introducing state and control reference in the criteria function.

$$V = \sum_{k=0}^{N-1} \begin{Bmatrix} [x(k) - x_{ref}(k)]^T Q_1 [x(k) - x_{ref}(k)] + \\ [u(k) - u_{ref}(k)]^T Q_2 [u(k) - u_{ref}(k)] \end{Bmatrix} \qquad (20)$$

Where, Q_1 and Q_2 are the symmetric nonnegative definite matrices. The cost function should include all states of the system in Q_1 matrix. While matrix Q_2 should include all inputs of the system. The simple theory to choice the pole of matrix Q_2 to determines the steady state response of the system. If the pole is placed at origin i.e. near to zero, then optimal controller will be a dead beat controller and provide fast steady state response. Whereas, if the pole is placed away from origin then steady state response will be slower as compared to dead beat controller. The proposed weighted matrix of both Q_1 and Q_2 are as follow:

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } Q_2 = 0.2 \qquad (21)$$

After initializing F, G, Q_1 and Q_2 matrices in MATLAB, the feedback controller gain can be evaluated by applying numerical method offered by the MATLAB Optimization toolbox.

$$L = dlqr(F, G, Q_1, Q_2) \qquad (22)$$

$$L = [\,1.3458 \quad 0.2214 \quad 0.1274 \quad 0.2208] \qquad (23)$$

Fig. 3 shows the simulink model of designed LQG controller based on the estimated states. It can be observe from the separation principle; the observer together with a state space feedback gain can be separated into two separate problems.

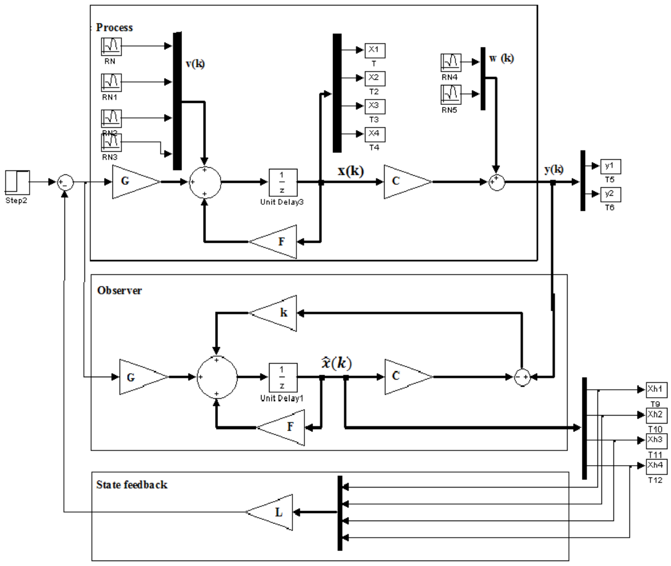

Fig-3: Simulink model of LGQ controller

V. RESULTS

A. Kalman Filter Simulation Results

The simulation result shown in Fig.4 are the estimated states of helicopter system using Kalman filtering observer. X1h is representing the estimated decline angle of helicopter, while X3h representing the estimated horizontal position. Fig. 5 shows the output results of both decline angle and horizontal position of the helicopter system denoted by Y_1(k) and Y_2(k) respectively. It can be seen from the figure that decline angle and horizontal position are inversely proportional to each other. As the decline angle is decreasing the helicopter is moving towards the horizontal direction. It is also noticeable that estimated states of both angle and position are very much similar to the value measured from the process output. Simulation results also proved the credibility of Kalman filter observer by comparing the estimated states such as position and angle with the model output.

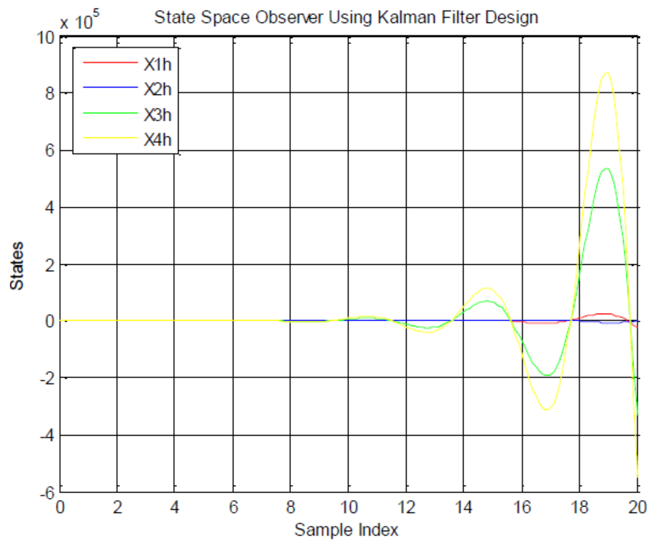

Fig-4: States representation of helicopter system using Kalman filter observer

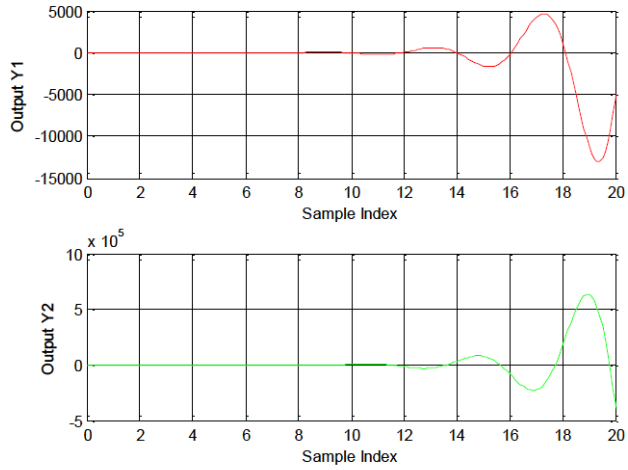

Fig-5: Process output of helicopter system

B. LQG Controller Simulation Results

Simulation results shown in Fig. 6, once again illustrates all system state, after implementing LQG controller based on Kalman filter observer. When LQG controller is applied to the specified helicopter model it goes toward stability. Now both position and the decline angle of the helicopter are in control. Here, reference signal X_{ref} is used to move helicopter from initial position at 0 to desire position at 100. It can be observed from Fig.7 that, decline angle of helicopter (Y_1) increases, reaching at the peak value around 10 and then decreased towards 0, before achieving the steady state at 2 second. During this period, helicopter moved gradually to the desire position (Y_2) at 100 and reaching at steady state value at sample interval 2 second, when exactly the helicopter decline angle becomes zero. The control signal shown in Fig.8, is the amount of decline of the rotor angle "δ" that force the helicopter to decline to achieve the desire horizontal position. The magnitude of control signal is very high because system is forced to move to desired position quickly.

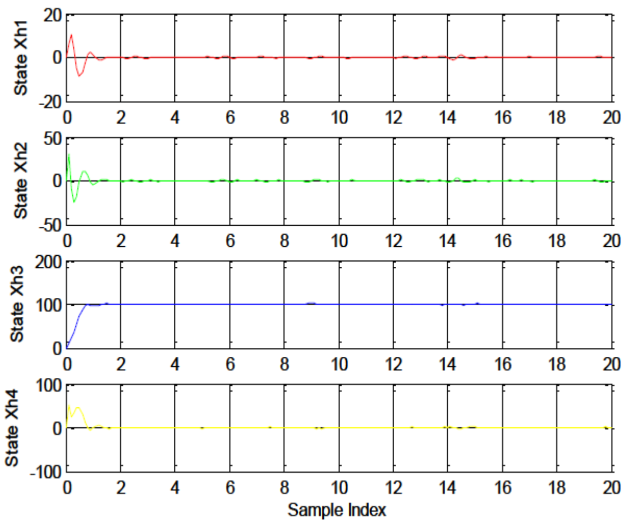

Fig-6: States representation of helicopter system using LQG controller

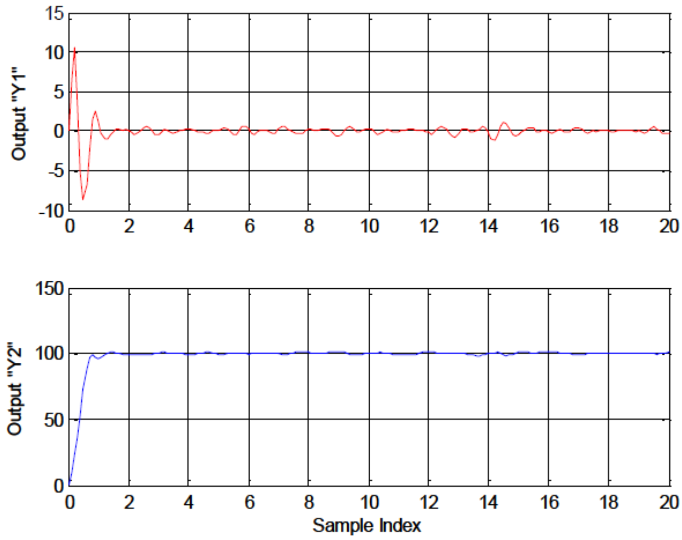

Fig-7: Output states representation of helicopter system using LQG controller

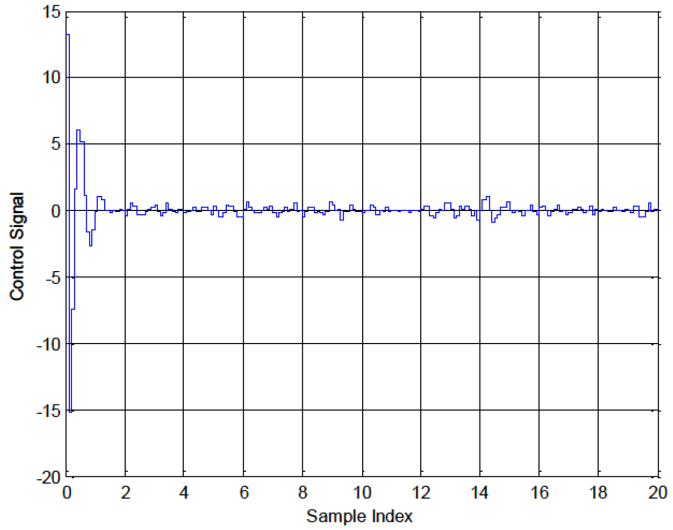

Fig-8: Control signal represents decline of the rotor indicated by $\boldsymbol{\delta}$

VI. CONCLUSION

This paper reviewed a non-linear motion of a helicopter, and proposed a Kalman based observer and Linear Quadratic Gaussian (LQG) based controller design. Only two-degree motion was considered that is necessary to steer the helicopter in a straight line. The mathematical foundation developed, and the subsequently testing using MATLAB proved that the proposed solution was able to successfully track the changing helicopter position. A future extension may possibly include the vertical motion as well.

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