Dynamic and Steady State Analysis of Induction Machine

Kriti, Jatinder Singh,Vivek Pahwa

Abstract--- In this paper, a model composed of fifth order differential equations of three-phase squirrel cage induction motor using two axis-theories in synchronously rotating reference frame, for dynamic/transient analysis is developed. The analysis has been extended to study the behaviour of three-phase induction machine during load changes and with different MOI's. The proposed model is implemented using well established and globally accepted software tool i.e matlab/simulink. Further, the transfer function of three-phase induction machine under consideration is developed for stability analysis. On the basis of analysis a specific recommendation has been given.

Index terms: Modeling, moment of inertia, (MOI), simulation, stability, three-phase induction machine.

I. INTRODUCTION

With the advancement in power electronics devices, ac drives, particularly induction machine drives are being preferred in contrast to dc drives, despite its precise speed control. During 1980's, lot of research has been carried out in the area of development of induction machine drive system for achieving better performance in terms of speed and torque characteristics. The performance analysis can be categorized as steady-state analysis, and dynamic analysis. For some purposes, steady-state analysis is found to be adequate, but during load change and starting with different MOI's, the dynamic analysis of the system is useful.

To predict transient behavior of induction motors [1] concluded that anyone of the three reference frames namely stator reference frame, rotor reference frame and synchronously rotating reference frame and can be used. [2] illustrated the effect of neglecting the moment of inertia of external system. [3] presented dq model of induction motor drives to analyze their effects on the dynamic response and stability of the system. [4] calculated poles and zeros using transfer functions approach for a frequency-controlled open-loop induction motor and concluded that the induction motor dynamic response differs according to the inputs and the motor load torque. [5] recommended specific tests to estimate machine parameters to proceed with dynamic simulation. So, the accurate determination of parameters of a machine is very important for carrying out various analyses. Also the performance of the machine changes significantly as inertia of the externally coupled system with the machine changes and as the load torque changes. The change of inertia of the machine and load torque on machine has great influence on the machine stability.

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Therefore, in this paper the application of matlab/simulink computer software is used for comparative analysis of experimental and simulated results. Further, the effect of change of moment of inertia and load torque on system performance is investigated.

II. MACHINE MATHEMATICAL MODEL

In this paper, the mathematical model of the induction motor is modeled using matlab / simulink. This model is described in space vector formulation in synchronously rotating reference frame associated with the frequency w_e of the stator excitation [3,7,8]. The three-phase stator voltages of an induction machine under balanced conditions can be expressed as:

$$V_a = \sqrt{2} v_s \cos(w_e t) \tag{1}$$

$$V_b = \sqrt{2}\, v_s \cos(w_e\, t - \tfrac{2\pi}{3}) \tag{2}$$

$$V_c = \sqrt{2}\, v_s \cos(w_e\, t + \tfrac{2\pi}{3}) \tag{3}$$

Where V_a,V_b & V_c are the bus bar voltages for phase a, b & c respectively. v_s is the rms voltage . w_e is the angular speed (rad/sec) in synchronously rotating reference frame.

2.1 THREE-PHASE TO TWO-PHASE CONVERSION:

To convert 3-phase voltages to voltages in the 2-phase synchronously rotating reference frame , they are converted to 2-phase stationary frame (αβ) using equation (4) and then from the stationary frame to the synchronously rotating frame (dq) using equation (5). In place of voltage and current linkage there may be currents or flux linkage:

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \left(\frac{\sqrt{2}}{3}\right) \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{4}$$

Then, the direct and quadrature axes voltages are:-

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \tag{5}$$

Where 'θ' is the transformation angle.

The final transformation becomes:

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} (\sqrt{2}3) \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{6}$$

2.2 TWO-PHASE TO THREE-PHASE CONVERSION

This conversion does the opposite of the abc-dq conversion for the current variables using (7) and (8) respectively by following the same implementation techniques as before

$$\begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = (\sqrt{2/3}) \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix} \quad (8)$$

Then the final transformation becomes

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} (\sqrt{2/3}) \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} I_d \\ I_q \end{bmatrix} \quad (9)$$

The voltage equations for the induction machine with currents as state variables in the synchronously rotating reference frame are given as:

$$\begin{bmatrix} v^e{}_{qs} \\ v^e{}_{ds} \\ v^e{}_{qr} \\ v^e{}_{dr} \end{bmatrix} = [x] \begin{bmatrix} I^e{}_{qs} \\ I^e{}_{ds} \\ I^e{}_{qr} \\ I^e{}_{dr} \end{bmatrix} \quad \text{Where [x] is given as} \quad (10)$$

$$\begin{bmatrix} R_1 + L_1 p & w_e L_1 & L_m p & w_e L_m \\ -w_e L_1 & R_1 + L_1 p & -w_e L_m & L_m p \\ L_m p & (w_s - w_e) L_m & R_2 + L_2 p & (w_e - w_r) L_2 \\ -(w_e - w_r) L_m & L_m p & -(w_e - w_r) L_2 & R_2 + L_2 p \end{bmatrix}$$

Where p is the differential operator. R_1, L_1 is stator phase resistance and inductance respectively. R_2, L_2 is rotor phase resistance and rotor leakage inductance, referred to stator respectively. L_m is magnetizing inductance. $v^e{}_{ds}$, $v^e{}_{dr}$ are the stator and rotor voltages aligned with the direct axis in synchronously rotating reference frame. $I^e{}_{ds}, I^e{}_{dr}$ are ttehe stator and rotor currents aligned with the direct axis in synchronously rotating reference frame, $v^e{}_{qs}$, $v^e{}_{qr}$ are the stator and rotor voltages aligned with the quadrature axis in synchronously rotating reference frame. $I^e{}_{qs}, I^e{}_{qr}$ are the stator and rotor currents aligned with the quadrature axis in synchronously rotating reference frame the electromagnetic torque is given by

$$T_e = (3/2)(P/2)\, L_m (I^e{}_{qs} I^e{}_{dr} - I^e{}_{ds} I^e{}_{qr}) \quad (11)$$

The voltages, currents,torque,stator frequency and rotor speed in their steady state are designated by subscript o in the variables and the perturbed increments are designated by a δ preceding the variables[10].

$$v^e{}_{ds} = v^e{}_{dso} + \delta v^e{}_{ds} \quad (12)$$

$$v^e{}_{qs} = v^e{}_{qso} + \delta v^e{}_{qs} \quad (13)$$

$$I^e{}_{qs} = I^e{}_{qso} + \delta I^e{}_{qs} \quad (14)$$

$$I^e{}_{ds} = I^e{}_{dso} + \delta I^e{}_{ds} \quad (15)$$

$$I^e{}_{qr} = I^e{}_{qro} + \delta I^e{}_{qr} \quad (16)$$

$$I^e{}_{dr} = I^e{}_{dro} + \delta I^e{}_{dr} \quad (17)$$

$$T_e = T_{eo} + \delta T_e \quad (19)$$

$$T_l = T_{lo} + \delta T_l \quad (20)$$

$$w_s = w_{so} + \delta w_s \quad (21)$$

$$w_r = w_{ro} + \delta w_r \quad (22)$$

$$T_{eo} = (3/2)\,(P/2)\, L_m (I^e{}_{qso} I^e{}_{dro} - I^e{}_{dso} I^e{}_{qro} \quad (23)$$

$$P[x] = A[X] + B[U] \quad (24)$$

Where $[x] = [\delta I^e{}_{qs}, \delta I^e{}_{ds}, \delta I^e{}_{qr}, \delta I^e{}_{dr}]^t$

$[U] = [\delta v^e{}_{qs}, \delta v^e{}_{ds}, \delta v^e{}_{qr}, \delta v^e{}_{dr}, \delta w_s, \delta T_l]t$

$A = P_1^{-1} Q_1$

$B = P_1^{-1} R_1$

$$P_1 = \begin{bmatrix} L_1 & 0 & L_m & 0 & 0 \\ 0 & L_1 & 0 & L_m & 0 \\ L_m & 0 & L_2 & 0 & 0 \\ 0 & L_m & 0 & L_2 & 0 \\ 0 & 0 & 0 & 0 & J \end{bmatrix} \quad (25)$$

$k_2 = (3/2)\,(P/2)^2\, L_m$

$Q_1 =$

$$\begin{bmatrix} -R_1 & -w_{so} L_1 & 0 & -w_{so} L_m & 0 \\ w_{so} L_1 & -R_1 & w_{so} L_m & 0 & 0 \\ 0 & -(w_{so} - w_{ro}) L_m & -R_2 & w_{ro} L_2 & I^e{}_{dso} + L_r I^e{}_{dro} \\ (w_{so} - w_{ro}) L_m & 0 & (w_{so} - w_{ro}) L_m & -R_2 & -(L_m I^e{}_{qso} + L_r I^e{}_{qro}) \\ k_2 I^e{}_{dro} & -k_2 I^e{}_{qro} & -k_2 I^e{}_{dso} & k_2 I^e{}_{qso} & -B \end{bmatrix}$$

(26)

$R_1 =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -L_1 I^e{}_{dso} + L_m I^e{}_{dro} & 0 \\ 0 & 1 & 0 & 0 & L_1 I^e{}_{dso} + L_m I^e{}_{dro} & 0 \\ 0 & 0 & 1 & 0 & -L_m I^e{}_{dso} + L_r I^e{}_{dro} & 0 \\ 0 & 0 & 0 & 1 & L_m I^e{}_{dso} + L_r I^e{}_{dro} & 0 \\ 0 & 0 & 0 & 0 & 0 & -P/2J \end{bmatrix}$$

(27)

III. RESULTS AND DISCUSSION

With the help of conventional tests performed on 3hp, 415 volts, 50 hz, 1440rpm, 4pole three-phase induction motor the parameters calculated are as follows:

Per-phase stator resistance, R_1= 4.02 ohm.

Per-phase rotor resistance referred to stator side, R_2 = 2.6 ohm.

Per phase stator reactance, X_1 = 4.1 ohm. [$X_1 = 2\pi f L_1$].

Per-phase rotor reactance referred to stator side, X_2 = 4.1 ohm. [$X_2 = 2\pi f L_2$]

Per-phase mutual reactance Xm =87 ohms

Moment of inertia, J= 0.1 kg-m^2.

The results of laboratory experiments performed on 3hp, 415volts, and 50Hz three-phase induction motor is compared with those obtained with computer simulation of 3hp, 415volts, and 50Hz three-phase induction motor using matlab/simulink. The model is compared for stator current with changes in load torque. As the load torque increases the stator current increases. On comparison (refer figure 3.1) it is observed that there is close similarity between experimental and simulated results.

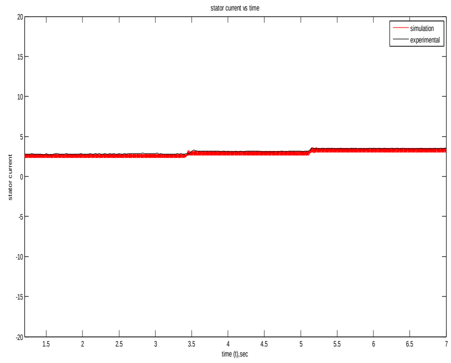

Figure 3.1 Comparison of simulated and experimental results.

3.1. DYNAMIC SIMULATION

The dynamic simulation considers the instantaneous effects of varying voltages/currents stator frequency and torque disturbances. Thus, in this section the dynamic behavior of induction motor for dynamic analysis in synchronously rotating reference frame is analyzed during no-load with different values of MOI's and load torques [2,7].

The effects of simulation of three-phase induction motor during no load condition are as follows:

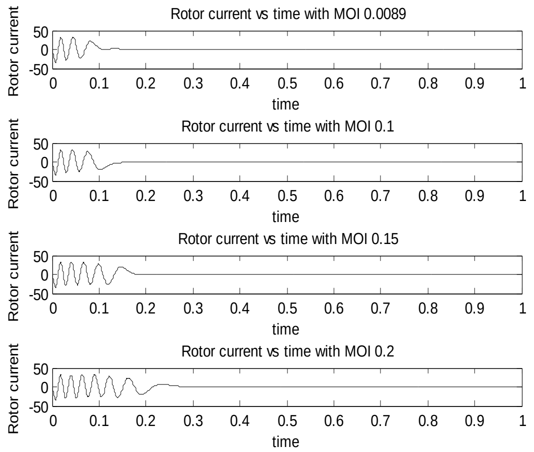

Figure 4.1 Rotor current vs time

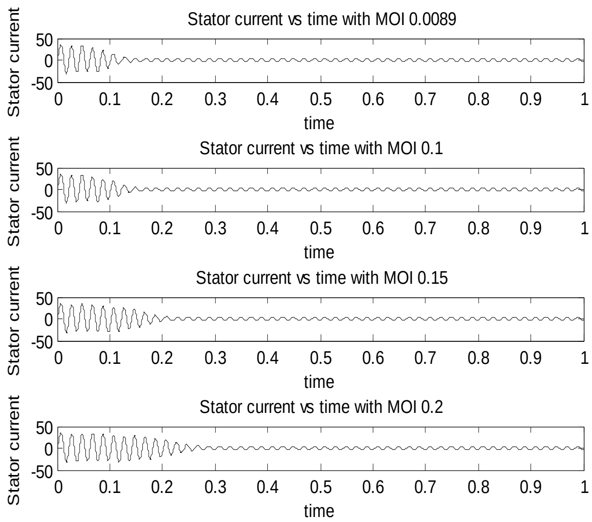

Figure 4.2 Stator current vs time

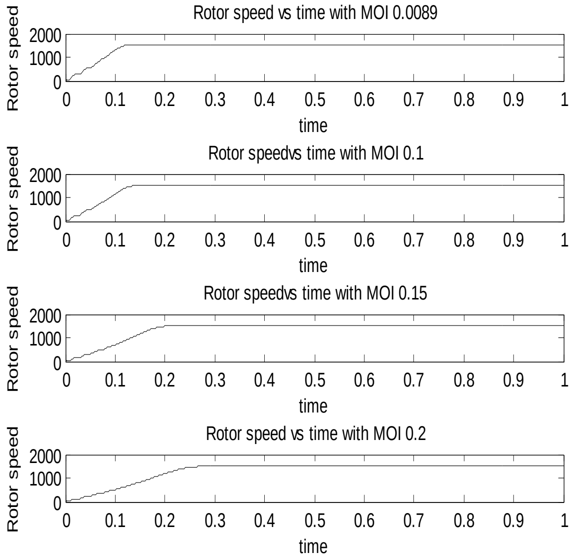

Figure 4.3 Rotor speed vs time

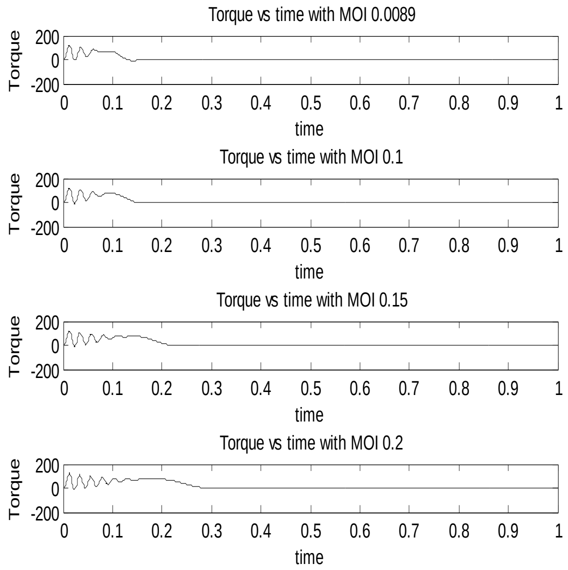

Figure 4.4 Torque vs time

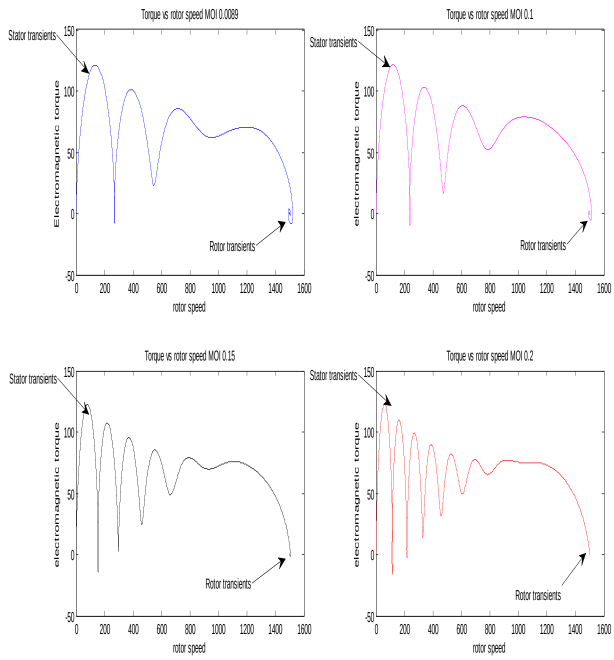

Figure 4.5 Rotor speed vs torque

As the friction and windage losses are neglected the machine accelerates to synchronous speed which is 1500 rpm. Low values of MOI results in to a low settling time, whereas it is increasing with increase in MOI as shown in figure 4.1 To 4.5. Speed build up is found to be smooth with large value of MOI as shown in figure 4.3. It can also be observed from torque speed characteristics refer figure 4.5 that on increasing the moment of inertia (MOI), the motor reaches its final operating condition without oscillations. After the transients associated with the stator circuit has subsided, the transients associated with the rotor circuit arises which cause the motor to reach final operating point in an oscillatory manner. The oscillations decreases with increase in moment of inertia, as shown in figure 4.5. It is interesting to note that at higher inertia the speed pulsation will be highly attenuated [6].

The effects of simulation with different load torque on three-phase induction motor are as follows:

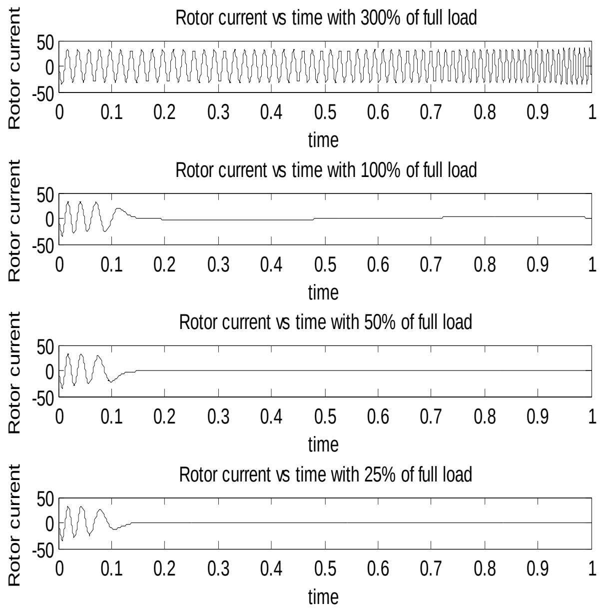

Figure 4.6 Rotor current vs time

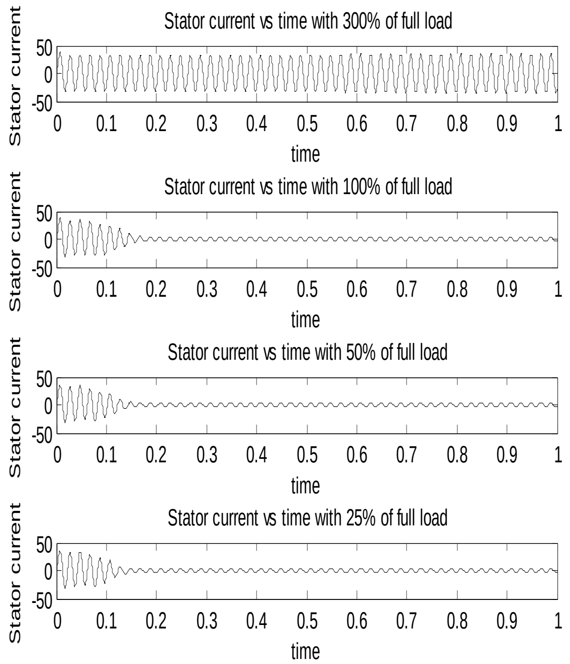

Figure 4.7 Stator current vs time

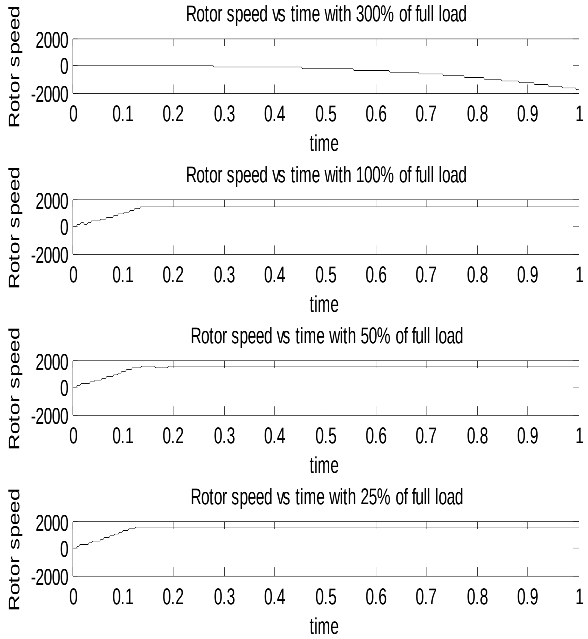

Figure 4.8 Rotor speed vs time

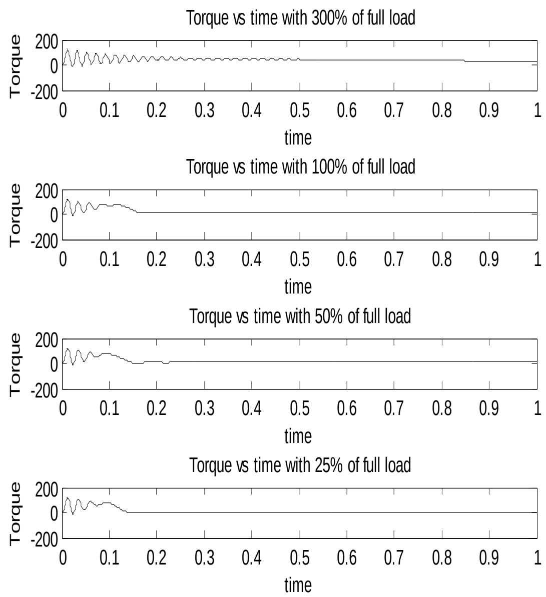

Figure 4.9 Torque vs time

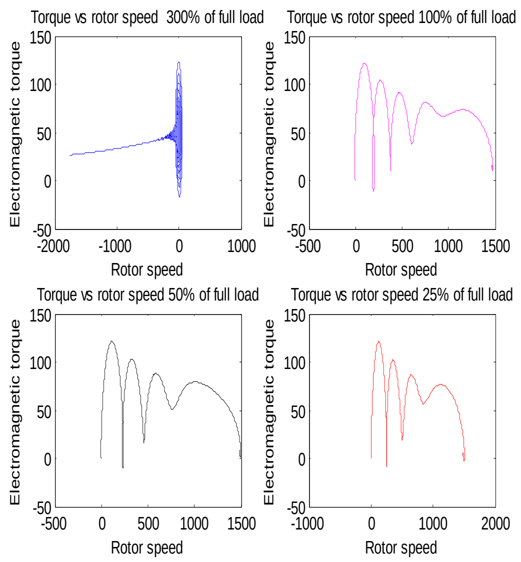

Figure 4.10 Torque vs Rotor speed

Low values of load torque results in to a low settling time, whereas it is increasing with increase in load torque as shown in figure 4.6 to 4.10. The dynamic performance during load torque changes is strongly influenced by the rotor electrical transients which causes the motor to exhibits damped oscillations about new operating point. At higher torque the electromagnetic torque speed characteristics are better damped refer figure 4.10. At high load torque the motor departs from stability as shown in figure 4.6 to 4.10. The stator current does not settle down and speed becomes negative [6].

3.2. STEADY STATE ANANLYSIS

To perform small-signal stability analysis of the induction motor, the eigen values of the system matrix [A] is determined in addition to the values of constant matrices [B], [C], and [D] if desired. matlab/simulink program is used here to perform such an analysis. At the beginning, the steady state of the simulink system of the induction motor is determined at some desired operating point using the trim function. When the initial values are obtained,then linmod matlab function is used to determine the [A, B, C, D] matrices of the small-signal model of the non-linear system about the evaluated steady-state operating point. Then the transfer function of the motor under consideration is determined and poles of the motor are calculated.The perturbation taken here is by changing the mechanical load torque between rated and above rated load [6-12]. Table 1 present the poles under such stability analysis. For our study, the input-output pair chosen is:

Input - Output
Mechanical Torque (Tmech,) - speed (Dwr/wb,)

Table 1: oles position with different load torques

Poles Position of Induction Motor at different Torques					
Poles Position	Torque(Nm) (From Graph)	Poles Position	Torque(Nm) (From Graph)	Poles Position	Torque (From Graph)
193.6000 +289.1000i	-14.2000 (Rated Torque,Tb)	-208.15 + 263.25i	-39.7476 (2.7991* Rated Torque,Tb)	-208.3000+ 263.4000i	-42.2000 (2.9718* Rated Torque, Tb)
193.6000-289.1000i		-208.15 – 263.25i		-208.3000-263.4000i	
15.1800+157.4000i		-51.31 + 178.28i		-51.2400+ 178.6000i	
15.1800-157.4000i		-51.31 – 178.28i		-51.2400- 178.6000i	
-101.4000		**0.0000**		**0.0719**	

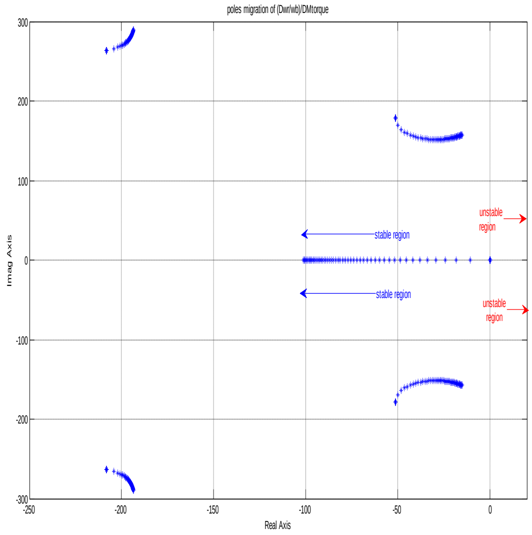

Figure 5.1 Pole Migration Plot

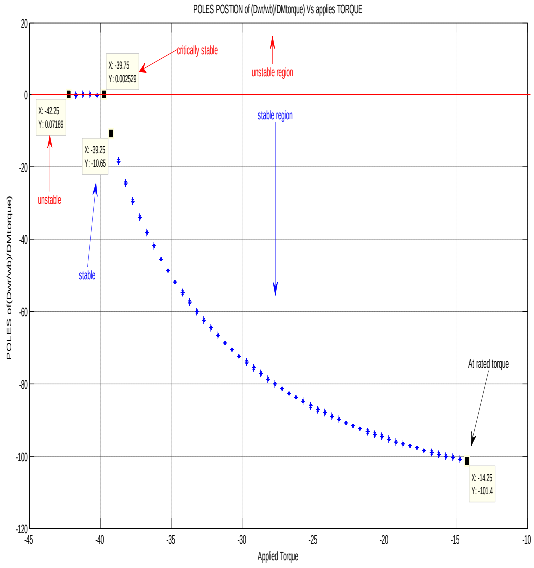

Figure 5.2 Poles vs applied torque

It can be observed from figure 5.1and 5.2 that as the motor under consideration was loaded with 2.7991 times the base load torque i.e.14.2 Nm the motor is critically stable and the pole related to torque lies in stable region. A slight increase in load torque pushes the motor in unstable region i.e. pole related to torque shifts to positive region. Under heavy loading the motor departs from stability. as shown in figure 5.1and 5.2. The motor remains stable as far as the loading is within the limits of the motor's developed torque. On the basis of analysis it is recommended that the motor will be stable up to 2.7991 times the base load torque i.e.14.2Nm and after that machine will move towards unstable region [5, 6]. Dynamic and Steady State Analysis of Induction Machine

IV. CONCLUSION

Matlab/Simulink based model gives the better understanding of dynamic behaviour of induction motor. Comparison of simulated results of dynamic model with experimental results proves the validity of model used. Conventional tests are used to determine various parameters of three-phase induction motor. On increasing the moment of inertia (MOI) the motor reach its final operating condition without oscillations.. On increasing the moment of inertia (MOI) the motor becomes more stable. On increasing the load torque the motor response is more damped. The motor remains stable as far as the loading is within the limits of motor developed torque. Under heavy loading the motor departs from stability. Further, the effect of saturation can also be included for accurate analysis of induction machine, as the concept of saturation is never ending area of research. Fault analysis of three-phase induction motor can also be studied.

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