Extended Kalman Filter Based State Estimation of Stepper Motor

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Abstract— ***State estimation process is one of the major concerns for controlling and monitoring systems in industry which requires high-cost measurements or involves unmeasurable variables of nonlinear systems. These drawbacks can be highly eliminated by designing systems without using any kind of sensors. In the proposed work, the state estimation technique is used for the state estimation of stepper motor. The theoretical basis of Extended Kalman Filter algorithm is explained in detail and its performance is tested with simulations.***

A stochastically nonlinear state estimator named Extended Kalman Filter is presented. The motor model designed for EKF application involves rotor speed, rotor position and stator currents of the stepper motor. Thus, by using this estimator the states of the stepper motor can be estimated.

Index Terms—Extended Kalman Filter, non linear system, state estimation, stepper motor

I. INTRODUCTION

Most mechanical movements are performed by using electrical motors in almost every industrial process. In industry, the electrical motors consume a large percentage of the produced electrical energy. The control of the electrical motors plays an important role in the continuous increase of electrical energy consumption. The electrical motors operate at constant and variable speeds. Recent advances in technology support the evolution of the electrical machine drive systems. The motor drive systems controlled at variable speeds are widely used in industry. An effective speed control may be achieved by using a closed-loop control system, where motor current, position and speed of the rotor, etc., need to be known. All these requirements lead to an increase in the total cost of the motor drive systems. Generally, the speed/position information of motor can be directly measured by using an encoder or a tacho generator which are mounted on the motor shaft.

However, the cost and volume of the system, weight of motor, and hardware complexity are increased by using a shaft-mounted measurement. Therefore, the reliability of drive system is reduced particularly at hard work environments [1], [2]. To reduce the cost of the system and increase its robustness and reliability, the position and speed of the rotor can be estimated by using the observers. The observer is an algorithm and consists of mathematical state equations. In motor drive applications, the observers estimate the position/speed of motor without using any kind of shaft speed sensors. The observers are more reliable than the sensored operations [3]–[5]. However, the observers require some mathematical equations and some measurements of motor such as current and voltage. The speed information must be estimated at high accuracy. Any variable of linear state-space system can be estimated by using the KF which is a basic recursive solution method. However, the KF cannot be directly applied to estimate a state variable of nonlinear dynamic systems. In a nonlinear dynamic system, the EKF can be used instead of the KF. It is the nonlinear version of the KF which linearizes about the current mean and covariance. In the EKF, the state transition and observation models need not be linear functions of the state but may instead be differentiable functions [6], [7]. However, the EKF always approximates the process and observation noises to be Gaussian. Speed estimation methods for stepper motor control are becoming very popular in recent years. Elimination of the speed sensors and the associated measurement cables have the advantages of lower cost, ruggedness, as well as increased reliability. State estimation is the process of estimating the values of parameters based on measured data having random component. The parameters explain the underlying physical setting in such a way that their value affects the distribution of the measured data. An estimator attempts to approximate the unknown parameters using the measurement data. Many types of estimators are available. The commonly used effective estimator is the Kalman Filter and its types. The Kalman Filter has a good dynamic behavior and disturbance resistance when compared to a nonlinear observer. It can work even under standstill conditions. The Kalman Filter provides optimal filtering of the noises in measurement and inside the system if the covariances of these noises are known. The Extended Kalman Filter (EKF) based on the nonlinear stepper model that includes the rotor speed, rotor position and stator currents as the state variables is presented in this paper.

The format of paper presentations is as follows: In Section II, the system model of stepper motor is given. In Section III, the nonlinear state estimation by using EKF is introduced. In Section IV, simulation implementations are evaluated for the performances of the EKF. Finally, the concluding remarks are stated in Section V.

II. MATHEMATICAL MODELLING OF STEPPER MOTOR

The stepper motor is an electromagnetic device that converts digital pulses into mechanical shaft rotation.

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The stator and rotor of the two phase stepper motor is shown in the Fig. 1. When the windings of the phase are energized, a magnetic dipole is generated on the stator side.

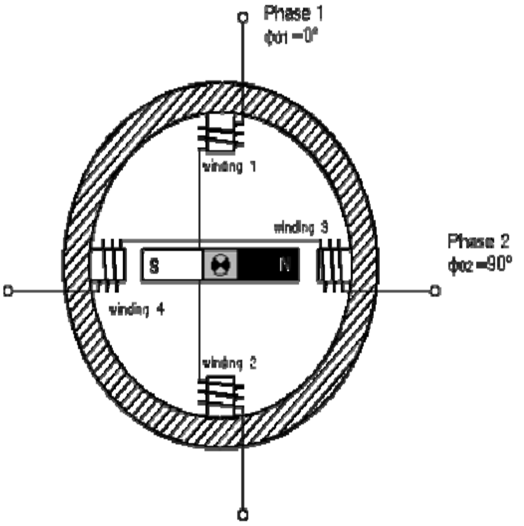

Fig. 1: Two Phase Stepper Motor

In order to estimate the stator currents, rotor speed and rotor position of the stepper motor, the following state equations of the stepper motor are needed.

$$\dot{I}_a = \frac{-R}{L} I_a + \frac{\omega\lambda}{L}\sin\theta + \frac{u_a + \Delta u_a}{L} \tag{1}$$

$$\dot{I}_b = \frac{-R}{L} I_b + \frac{\omega\lambda}{L}\cos\theta + \frac{u_b + \Delta u_b}{L} \tag{2}$$

$$\dot{\omega} = \frac{-3\lambda}{2J} I_a \sin\theta + \frac{3\lambda}{2J} I_b \cos\theta - \frac{F\omega}{J} + \Delta\alpha \tag{3}$$

$$\dot{\theta} = \omega \tag{4}$$

$$y = \begin{bmatrix} I_a \\ I_b \end{bmatrix} + \begin{bmatrix} v_a \\ v_b \end{bmatrix} \tag{5}$$

where I_a and I_b are the currents in the two motor windings respectively. θ and ω are the angular position and velocity of the rotor. R and L are the motor winding's resistance and inductance. λ is the flux constant of the motor. F is the coefficient of viscous friction that acts on the motor shaft and its load. J is the moment of inertia of the motor shaft and its load. u_a and u_b are the voltages that are applied across the two motor windings. Δu_a and Δu_b are noise terms due to errors in u_a and u_b. Δα is a noise term due to uncertainty in the load torque. y is the measurement. The two winding currents are measured by using sense resistors.

The measurements are distorted by measurement noises v_a and v_b, which are due to things like sense resistance uncertainty and electrical noise. In order to apply EKF to the motor, the states of the system have to be defined. The states can be seen by looking at the system equations and noting wherever a derivative appears. If a variable is differentiated in the system equations, then that quantity is a state. So we see from the above motor equations that our system has four states, and the state vector x can be defined as

$$x = \begin{bmatrix} I_a \\ I_b \\ \omega \\ \theta \end{bmatrix} \tag{6}$$

The system equation is obtained by discretizing the differential equations to obtain

$$x_{k+1} = f(x_k, u_k) + w_k \tag{7}$$

$$x_{k+1} = x_k + \begin{bmatrix} \frac{-Rx_k(1)}{L} + \frac{x_k(3)\lambda\sin x_k(4)}{L} + \frac{u_{ak}}{L} \\ \frac{-Rx_k(2)}{L} + \frac{x_k(3)\lambda\cos x_k(4)}{L} + \frac{u_{bk}}{L} \\ \frac{-3\lambda x_k(1)\sin x_k(4)}{2J} + \frac{-3\lambda x_k(2)\sin x_k(4)}{2J} - \frac{Fx_4(3)}{J} \\ x_k(3) \end{bmatrix}\Delta t + \begin{bmatrix} \frac{\Delta u_{ak}}{L} \\ \frac{\Delta u_{bk}}{L} \\ \Delta\alpha \\ 0 \end{bmatrix}\Delta t \tag{8}$$

$$y_k = h(x_k) + v_k \tag{9}$$

$$y_k = \begin{bmatrix} x_k(1) \\ x_k(2) \end{bmatrix} + \begin{bmatrix} v_{ak} \\ v_{bk} \end{bmatrix} \tag{10}$$

The derivative matrices are given by the

$$C_k = h'(\hat{x}_k) \tag{11}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{12}$$

$$A_k = \begin{bmatrix} -R/L & 0 & \lambda\sin\hat{x}_k(4)/L & \hat{x}_k(3)\lambda\cos\hat{x}_k(4)/L \\ 0 & -R/L & -\lambda\cos\hat{x}_k(4)/L & \hat{x}_k(3)\lambda\sin\hat{x}_k(4)/L \\ -3\lambda\sin\hat{x}_k(4)/2J & 3\lambda\cos\hat{x}_k(4)/2J & -F/J & -3\lambda[\hat{x}_k(1)\cos\hat{x}_k(4)+\hat{x}_k(2)\sin\hat{x}_k(4)]/2J \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (13)$$

The measurement noise terms, v_{ak} and v_{bk} are zero-mean random variables with standard deviations equal to 0.1 amps. In discrete time, the control inputs are equal to

$$u_{ak} = \sin 2\pi k\Delta t \quad (14)$$

$$u_{bk} = \cos 2\pi k\Delta t \quad (15)$$

The voltages that are applied to the winding currents are equal to these values plus Δu_{ak} and Δu_{bk}, which are zero-mean random variables with standard deviations equal to 0.001 amps. The noise due to load torque disturbances $\Delta\alpha_k$ has a standard deviation of 0.05 rad/sec^2. The values of the motor parameters are shown in the Table 1. These state equations are used by the Extended Kalman Filter to estimate the stator currents, rotor speed and rotor position.

Table 1: Parameters of the Stepper Motor

S.No	Motor parameters	Values
1	Winding resistance	1.9
2	Winding inductance	0.003
3	Motor constant	0.1
4	Moment of inertia	0.0001 8
5	Coefficient of viscous friction	0.001

III. EKF ALGORITHM

The EKF is a mathematical algorithm which can be used to estimate unmeasurable state variables of the system by using measured variables and statistic of noise. The EKF is a stochastic observer for recursive state estimation of a nonlinear dynamic system in real time by using noisy measured signals. The noise correlates with measurement and modelling inaccuracies. The EKF algorithm has two main stages, namely, prediction step and update (filtering) step. In the prediction step, a mathematical model of the system containing the previous estimates is used, and in the update step, a feedback correction scheme is used for continuously correcting the predicted states. The feedback update scheme needs an additional term to the predicted states, which contains the weighted difference of the measured and estimated output signals. However, the mathematical dynamic model must be well known for accuracy. Furthermore, the initial values of the covariance matrices must be arranged correctly. These can be obtained by considering the stochastic properties of the corresponding noises. Since these are usually not known, in most cases, they are used as weight matrices, but it should be noted that, sometimes, simple qualitative rules can be set up for obtaining the covariances of the noise vectors.

A critical part of the EKF is to use correct initial values for various covariance matrices namely, Q, R, and P. These have important effects on the filter stability and convergence time. The system noise covariance Q accounts for the model inaccuracy, the system disturbances, and the noise introduced by the voltage measurements (sensor noise and A/D converter quantization). The noise covariance R accounts for measurement noise introduced by the current sensors and A/D quantization [8]. Steps and initialization values of the EKF are presented as follows:

$$x(t) = A_d * x(t-1) + B_d * u(t-1) \quad (16)$$

where x(t) is the predicted state mean

$$P(t) = A_d * P(t-\mathbf{1}) * A_d^T + Q \quad (17)$$

where P(t) is the predicted state covariance

$$IM = C_d\, x(t) \quad (18)$$

where IM is the mean of the predictive distribution of y(t)

$$IS = R + C_d * P(t) * C_d^T \quad (19)$$

where IS is the covariance or predictive mean of y(t)

$$K = P(t)*C_d^T * IS^{-1} \quad (20)$$

where K is the computed Kalman gain. The updated state mean and the updated state covariance are given in (11) and (12) respectively

$$x(t) = x(t-1) + K * (y(t) - IM) \quad (21)$$

$$P(t) = P(t-1) - K * IS * K^T \quad (22)$$

The steps in one cycle of the algorithm are given from (16)–(22). The discrete time t is increased by one in every cycle.

IV. SIMULATION RESULTS

The Extended Kalman Filter algorithm is used to estimate the values of the current state variables such as stator currents, rotor speed and rotor position by using the values of previous state values and the nature of statistics of noise.

The estimated outputs for the stepper motor variables are shown in the Fig. 2, 3, 4 and 5.

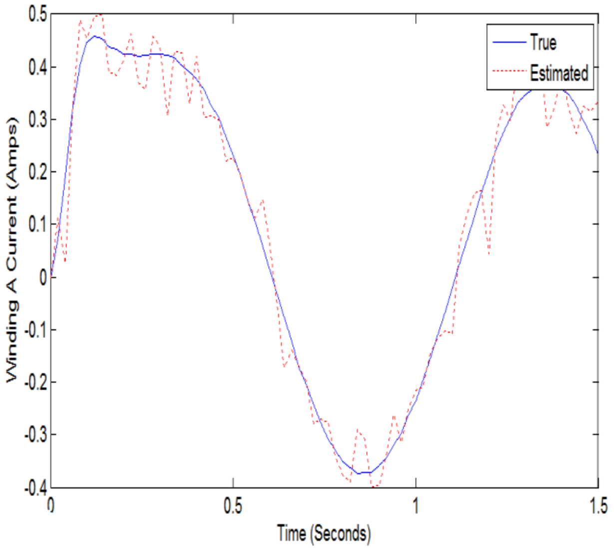

Fig. 2: Plots of estimated and true values of winding A current

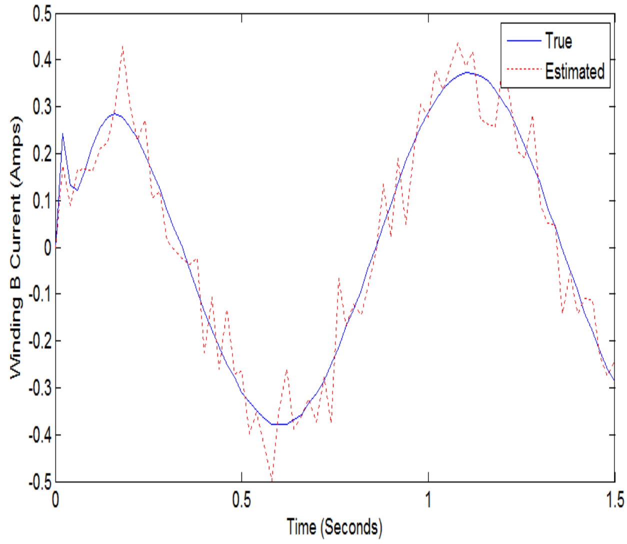

Fig. 3: Plots of estimated and true values of winding B current

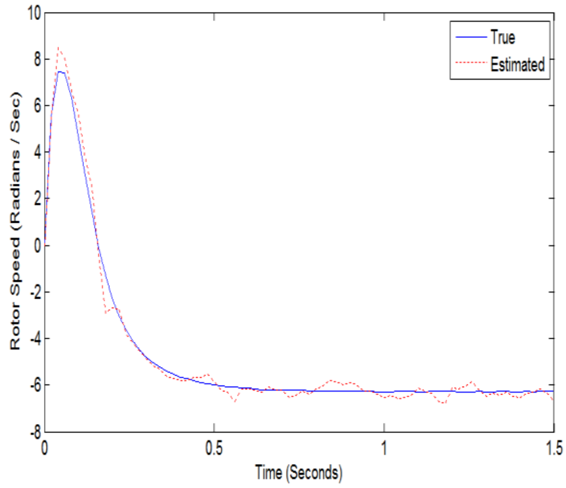

Fig. 4: Plots of estimated and true values of rotor speed

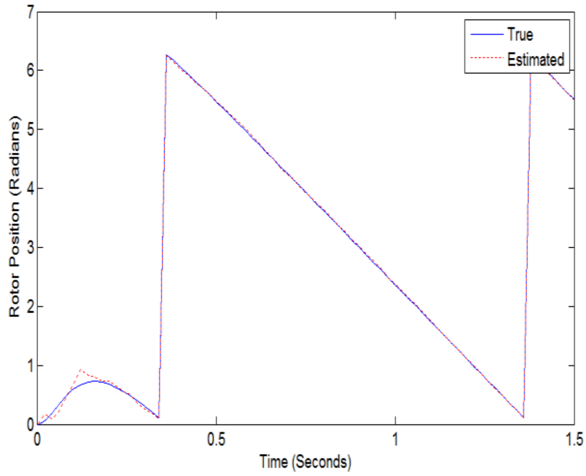

Fig. 5: Plots of estimated and true values of rotor position

In the Fig. 2 to 5, the straight line shows the true values of the stator currents and the dotted line shows estimated values by using the Extended Kalman Filter. It is found that the standard deviations of estimation error for both the estimated stator currents are 0.048857 and 0.050246 respectively.

The rotor speed and rotor position are estimated with the standard deviations of estimation error are 0.21378 and 0.010227 respectively as shown in the Figures 4 and 5.

V. CONCLUSION

The applications of stepper motors have grown significantly in recent years in the appliance industry and the automotive industry, among others. These motors are used in a variety of industries, including high and low propulsion technology, computer peripherals, machine tools, robotics, etc. Sensorless stepper motors are preferable to encoder-based systems because of compactness, low cost, low maintenance, and high reliability. The conventional sensorless method based on a neutral motor point has limited application since it has a low speed range, suffers from high common mode voltage noise and exhibits high frequency switching noise. So the Extended Kalman Filter is used to estimate the stator currents, speed and position of the stepper motor using its mathematical model.

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