An EOQ Model for Deteriorating Items with Quadratic Demand and Time Dependent Holding Cost

R.Amutha, E.Chandrasekaran

Abstract— This paper presents an inventory model for deteriorating items with Quadratic demand. An Exponential distribution is used to represent the distribution of time to deterioration. Shortages are not allowed and holding cost is time dependent. Our objective is to minimize the total cost. Numerical Examples is given to illustrate the solution procedure.

Keywords — Deterioration, Demand, holding cost, Inventory, Quadratic Demand, Shortages.

I. INTRODUCTION

In the past many researchers developed inventory control models Md Azizul Baten and Anton Abdulbasah Kamil [14]also developed an inventory model without shortage. The production cost, deteriorates, demands rate and holding cost do not vary for the model and also there is a continues demand for it.

Garima Garg, Bindu Vaish and Shalini[11] Gupta developed a model based on demand and production rate without any shortages . This model has additional features such as disposal cost and sales revenue.

There were models prepared with shortages and also without shortages .Many authors dealt with models for trade credit where the retailers were given some credit periods to settle the account. Hardik Soni, Nita H. Shah and Chandra K.Jaggi[3] colony has published a review about this credit. Ruxian Li, Hongjie Lan, John R. Mawhinney [13] also published a review on deterioration inventory models.

Demands coming under linear type, exponentially increasing or decreasing, function selling prize and production based were also discussed.

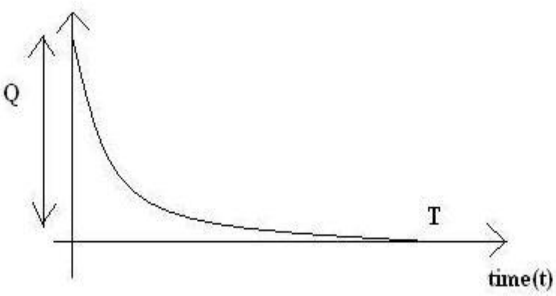
R.Begum S.K.Sahu & R.RSahoo [8]developed model with time dependent on Quadratic demand rate. Chaintanya kumar Tripathy and Umakanta Mishra[5]developed another model with Quadratic demand when the deterioration rate depends on two parameter Weibull distribution without shortages.

We have developed an inventory model for Quadratic demand when the deterioration rate follows an exponential distribution with time dependent holding cost and shortage not allowed. This model is illustrates with numerical example.

II. BASIC MODEL ASSUMPTIONS AND NOTATIONS

A. The inventory system deals with single item
B. The demand rate is assumed to be $D(t) = a+bt+ct^2$; the annual demand as a function of time where $a > 0$, $b > 0$and $c > 0$.
C. The lead – time is zero.
D. A finite planning horizon is assumed.
E. Shortages are not allowed.
F. A: set up cost per order
G. I (t): inventory level of the product
H. Holding cost is linear function of time $H(t) = \alpha + \beta t$, $\alpha > 0$, $\beta > 0$.
I. C_1 the deterioration cost
J. θ:the deterioration rate

III. DEVELOPMENT OF MODEL AND SOLUTIONS



Let I(t) be the on-hand inventory at any instant of time t ($0 \le t \le T$). The rate of change of inventory level is governed by the following differential equation:

$$\frac{dI(t)}{dt} + \theta I(t) = -(a+bt+ct^2), \quad 0 \le t \le T \qquad (1)$$

With boundary conditions I (T) = 0 & I (0) = Q Solving Equation (1)

$$I(t) = a(T-t) + \frac{(b+a\theta)(T^2-t^2)}{2} + \frac{(c+b\theta)(T^3-t^3)}{3} + \frac{c\theta(T^4-t^4)}{4} + \frac{c\theta^2(T^5-t^5)}{5} \qquad (2)$$

i. Set up cost: $\frac{A}{T}$

ii. Inventory holding cost

$$HC = \frac{1}{T}\int_0^T (\alpha + \beta t) I(t)\, dt$$

$$HC = \frac{\alpha a T}{2} + \frac{\alpha(b+a\theta)T^2}{3} + \frac{\alpha c\theta T^4}{5} + \frac{\alpha(c+b\theta)T^3}{4} + \frac{\alpha c\theta^2 T^5}{6} + \frac{\beta a T^2}{6} + \frac{\beta(b+a\theta)T^3}{8} + \frac{c\theta\beta T^5}{12} + \frac{3\beta(c+b\theta)T^4}{20}$$

Order Quantity

Put I (0) = Q

$$Q = I(0) = aT + \frac{(b+a\theta)T^2}{2} + \frac{(c+b\theta)T^3}{3} + \frac{c\theta T^4}{4} + \frac{c\theta^2 T^5}{5}$$

Deterioration cost

$$DC = \frac{C_1}{T}\left[Q - \int_0^T D(t)\,dt\right]$$

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R.Amutha, Research Scholar, Department of Mathematics, Presidency College, henna, India.

Dr.E.Chandrasekaran, Associate Professor, Department of Mathematics, Presidency College**,** Chennai, India.\

$$DC = \frac{aC_1\theta T}{2} + \frac{b\theta C_1 T^2}{3} + \frac{cC_1\theta T^3}{4} + \frac{cC_1\theta^2 T^4}{5}$$

Total cost per unit time is

$$K(T) = \frac{1}{T}\,[HC + DC + SC]$$

$$= \frac{A}{T} + \frac{aC_1\theta T}{2} + \frac{b\theta C_1 T^2}{3} + \frac{cC_1\theta T^3}{4} + \frac{cC_1\theta^2 T^4}{5} + \frac{a\alpha T}{2} + \frac{\alpha(b+a\theta)T^2}{3} + \frac{\alpha c\theta T^4}{5} + \frac{\alpha(c+b\theta)T^3}{4} + \frac{\alpha c\theta^2 T^5}{6} + \frac{\beta a T^2}{6} + \frac{\beta(b+a\theta)T^3}{8} + \frac{c\theta\beta T^5}{12} + \frac{3\beta(c+b\theta)T^4}{20}$$

Our objective is to minimize the total cost per unit time K(T). The necessary condition for total cost K(T) to be minimize is $\frac{\partial K}{\partial T} = 0$ and $\frac{\partial^2 K}{\partial T^2} > 0$ for all T > 0.

We get

$$\frac{\partial K(T)}{\partial T} = \frac{-A}{T^2} + \frac{aC_1\theta}{2} + \frac{2b\theta C_1 T}{3} + \frac{3cC_1\theta T^2}{4} + \frac{4cC_1\theta^2 T^3}{5} + \frac{a\alpha}{2} + \frac{2\alpha(b+a\theta)T}{3} + \frac{4\alpha c\theta T^3}{5} + \frac{3\alpha(c+b\theta)T^2}{4} + \frac{5\alpha c\theta^2 T^4}{6} + \frac{2\beta a T}{6} + \frac{3\beta(b+a\theta)T^2}{8} + \frac{5c\theta\beta T^4}{12} + \frac{6\beta(c+b\theta)T^3}{10} = 0$$ and

$$\frac{\partial^2 K(T)}{T} = \frac{2A}{T^3} + \frac{2b\theta C_1}{3} + \frac{3cC_1\theta T}{3} + \frac{12cC_1\theta^2 T^2}{5} + \frac{2\alpha(b+a\theta)}{3} + \frac{12\alpha c\theta T^2}{5} + \frac{3\alpha(c+b\theta)T}{2} + \frac{10\alpha c\theta^2 T^4}{3} + \frac{\beta a}{3} + \frac{3\beta(b+a\theta)T}{4} + \frac{5c\theta\beta T^5}{3} + \frac{18\beta(c+b\theta)T^2}{10} > 0$$

IV. NUMERICAL EXAMPLE

Consider an inventory system with following parametric values in proper units [A,a,b,c, θ, α, β,C_1] = [1000, 25, 40, 20, 0.08, 0.5, 0.011, 1.5] in their respective units. Then we get T = 2.6175, K = 537.9676, Q =369.9206

V. SENSITIVE ANALYSIS

Table: Variation in deterioration rate 'θ'

θ	T	Q	K	DC	HC
0.02	2.8478	402.4278	498.4113	7.9341	139.3290
.045	2.7432	387.3210	515.5538	16.7422	134.2738
0.06	2.6866	379.36.84	525.3715	21.5352	131.6186
0.08	2.6175	369.9206	537.9676	27.4508	128.4729
0.1	2.5545	361.5251	550.0473	32.8975	125.6838

Table shows that when hazardous rate increases, automatically inventory holding cost, deterioration cost and total cost are increases.

VI. CONCLUSION

The model developed in this paper assumes demand of a product to be quadratic with respect to time and follows constant deterioration rate with time dependent holding cost. Shortages are not allowed. Different costs have been illustrated through the numerical example and sensitive analysis.

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AUTHORS PROFILE

R.Amutha, Assistant Professor, Department of Mathematics, Dr.MGR Educational & Research Institute University, Research Scholar, Presidency College, Chennai-05. (e-mail:amutha.raja77@gmail.com).

Dr.E.Chandrasekaran, Associate Professor, Department of Mathematics, Presidency College, Chennai-05. (e-mail: e_chandrasekaran@yahoo.com)