

# Nonlinear Digital PID Controller for Position Controlled Electric Drive Systems

Srikanth Mandarapu, Sreedhar Lolla, M.V.Suresh Kumar

**Abstract** — This paper discusses the implementation of Nonlinear digital PID controller for position controlled electric drive systems. The drawback with PD controller is that it produces a non aperiodic response, when it encounters a maximum torque limit. To overcome this drawback, the nonlinear PD controller is redesigned so to produce aperiodic response. The applicability of PD controller is limited only for the cases of reference input changes, while along with reference input, if disturbance inputs are also considered, the output results in a steady state error (S.S.E), which is proportional to the disturbance value. To minimize the S.S.E while producing a strong aperiodic response, it is proposed to implement a nonlinear digital PID controller.

The proposed scheme in this paper is compared with the linear mode, is implemented in MATLAB and from the obtained results its possible use, limitations and counter measures have been studied.

**Index Terms**— anti wind-up, non linear PID controller, PD controller, quantizer, S.S.E, torque limiter.

**List of symbols:**

- $\theta$  Measured position (rad)
- $\theta^*$  Reference position ( rad)
- $\Delta\omega$  Speed error =  $\omega^* - \omega$  (rad/sec)
- $\Delta\theta$  Speed error =  $\theta^* - \theta$  (rad)
- $\omega_{BW}$  Closed loop band width (Hz)
- J Inertia of the moving parts ( $\text{kgm}^2$ )
- B Friction coefficient
- $T_{ref}$  Reference torque (Nm)
- $T_L$  Load toque (Nm)
- $T_{em}$  Driving torque (Nm)
- $K_m$  Motor torque constant (Nm)
- $K_p$  Proportional gain.
- $K_i$  Integral gain.
- $K_d$  Derivative gain
- $T_s$  Sampling period (sec)
- $T_{max}$  Maximum value of the allowable torque (Nm)
- $\omega_{max}$  Maximum value of the allowable speed (rad/sec)

## I. INTRODUCTION

The case of motion control of electric drive systems are broadly classified as speed control and position control. For speed control it requires only one state variable as feedback. Whereas, position control requires two state variables as feedback, hence the implementation of speed control is easier than compared to position control.

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Also for the same reason the bandwidth of the output response with the case of speed control is more when compared to that with the case of position control. With the case of position controlled systems the performance is seriously affected both by the reference input and with disturbance input. Another issue seriously required for qualitative and quantitative performance of the position controlled drives is that, both the speed and torque must be limited within their max and min values. Whereas, it will be only torque that will be restricted within limits for the case of speed controlled systems.

PD controller for position control in its linear operation and its limitations with torque limiter are studied in [1], [4]. To overcome the limitations of PD controller when it encounters a torque limiter the implementation of PD controller in non linear mode is discussed in [3], [6]. Since for the reason that PD controller when designed in non linear mode could behave well for reference input variations only. And this design could produce a S.S.E when controller encounters a torque disturbance. When the PD is augmented with integrator, then the S.S.E will be well taken care and the behaviour of PID in its linear mode is studied in [2]. The study of windup with PID position controllers are discussed in [8]. The undesirable impact of arbitrary quantizer setting is discussed in [9]. Parameter settings for p,i,d are discussed in [6]. This paper deals with the implementation of PID which encounters non linear conditions considering the affects of both high heavy side input as reference and load disturbances. The analysis and design using simulink in MATLAB is studied based on [7], hence all the cases were studied and were simulated in MATLAB and so the pros and cons of each scheme were observed.

## II. NON LINEAR PD POSITION CONTROLLER WITH LOAD TORQUE DISTURBANCE

In order to avoid the overshoot in the response it is necessary to approach the target position in such a way that the speed of the motion reduces to zero as the output position reaches the target. Hence the PD position controller must be modified in such a way that the position error and the speed come to zero at the same time.

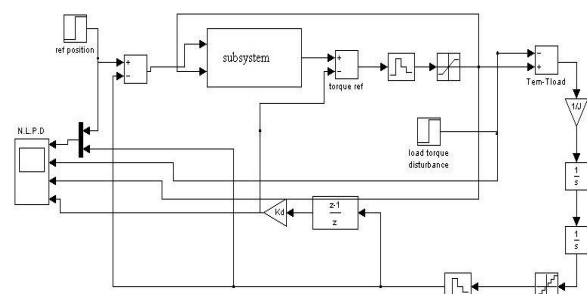


Fig .1 Simulation block diagram of position controlled electric drive system using non-linear PD controller.



It may be noted that incremental form of anti wind up mechanism generally used for speed control cannot be used with PD position control. Hence a suitable non linear form of PD controller is designed for producing aperiodic response.

Fig (1) shows that the block diagram of position controlled electric drive with a non linear PD controller. As observed from fig (6) in results, the response has a steady state error of 0.007 rad for a load torque disturbance of 5 N-m. This problem needs to be addressed.

It is known from [1], [4] that if integrator is introduced in the controller then the S.S.E resulting from heavy side disturbance can be made zero.

III. PARAMETER SETTING OF KP, KD, KI

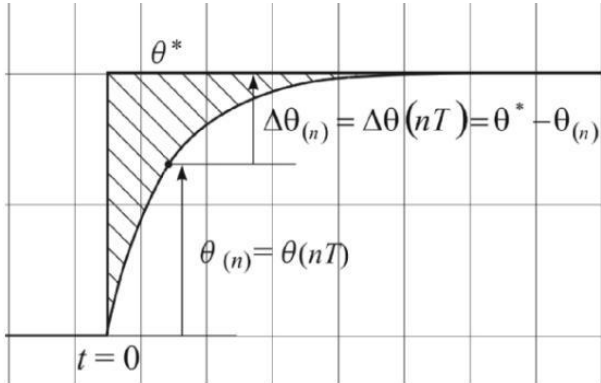


Fig .2 Typical aperiodic response of system for heavy side input.

The presence of oscillations in the step response depends on the damping factor of the polynomial zeros. These also determine the rise time and the closed loop bandwidth. With the position controller structure an arbitrary setting of the closed loop poles is not feasible.

Moreover, with the two state variables observed within the control object (omega, theta) and with the third one being the output of the error integrator, one would expect a characteristic polynomial of the third order. The existence of fourth hidden state comes from the intrinsic delay of the sampling period T.

The zeros a1, a2, a3, a4 are determined by the normalized value of the feedback gains p,d,i. the closed loop poles and feedback gains are tied by the following relations.

$$\begin{aligned}
 a_1 + a_2 + a_3 + a_4 &= 3-p-i-d \\
 a_1 a_2 + a_1 a_3 + a_1 a_4 + a_2 a_3 + a_2 a_4 + a_3 a_4 &= 3+i-d \\
 a_1 a_2 a_3 + a_1 a_2 a_4 + a_1 a_3 a_4 + a_2 a_3 a_4 &= 1+p+d \\
 a_1 a_2 a_3 a_4 &= d
 \end{aligned}
 \tag{1}$$

From the fig (2), it can be observed that, smaller the shaded area, the faster the step response given by

$$Q = \sum_{k=0}^{\infty} \Delta\theta(kT)
 \tag{2}$$

Expressing the value of Q in terms of p,d,i and after further substitutions one would end up with equation (3)

$$\begin{aligned}
 d_{opt} &= 0.216 \\
 p_{opt} &= 0.0516 \\
 i_{opt} &= 0.005219
 \end{aligned}
 \tag{3}$$

The optimized parameter setting minimizes the sum of the error samples in the step response and results in a spectrum of the closed loop poles which lies within the unit circle in the Z domain.

IV. LINEAR MODE PID POSITION CONTROLLER

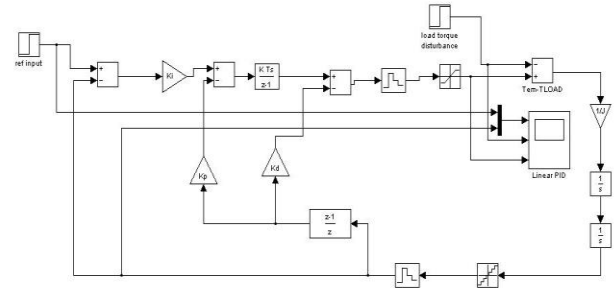


Fig .3 Simulation block diagram of position controlled electric drive system using linear PID controller.

Fig (3) shows the simulation block diagram of the PID controller which could operate for linear modes. It is worth mentioning that with the optimum parameter setting both PD and PID would give strictly aperiodic response but the rise time of the response produced by PD controller is low when compared with that produced by PID controller.

V. NON LINEAR PID POSITION CONTROLLER

Hence in this paper, it is proposed as following. With the case of linear position PID controller, if for any reason the tracking error Delta theta increases and brings the driving torque to the limit Tmax, the system enters non linear operating mode, resulting in poorly damped oscillations and eventually in-stability.

However in cases when the tracking increases and brings the system into non linear mode, it is necessary to provide control means and guarantee controllability of the system.

It is known from fig (2), that the speed of the system approaching the target has to be limited in proportion to the scheduled path Delta theta. The relationship between the residual speed and braking distance Delta theta is obtained as

$$\Delta\theta = \frac{J \omega^2}{2T_{max}}
 \tag{4}$$

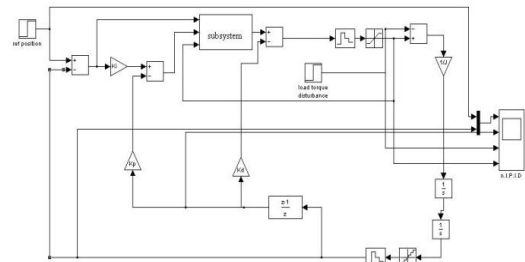


Fig .4 Simulation block diagram of position controlled electric drive system using the proposed nonlinear PID controller

This suggests that a position controlled system approaching the target at a speed of omega does not produce an overshoot provided that the remaining path Delta theta = theta\* - theta is atleast J\*omega^2/2Tmax. As has been said earlier, for qualitative and quantitative performance of position control, both the speed and the torque must be limited within the limits. From the equation (4), the speeds is limited to a value given by the relation

$$|\omega m| = \sqrt{\frac{2T_{max}|\Delta\theta|}{J}}
 \tag{5}$$



The speed limit for the cases when the load disturbances are small then from the equation (5),  $\omega_{max}$  can be calculated and given as

$$\omega_{max} \approx \frac{k_i \Delta\theta}{k_p T} \quad (6)$$

The largest error attainable in linear operating mode can be found from equations (4), (6) is given as

$$\Delta\theta_{max} = \frac{2T_{max}}{J} \left( \frac{k_p T}{k_i} \right)^2 \quad (7)$$

Equation (8) represents the max speed at the border between the linear and non linear mode.

$$\omega_{max} = \frac{2T_{max}}{J} \left( \frac{k_p T}{k_i} \right) \quad (8)$$

Fig (4) shows the simulation block diagram for implementation of non linear PID position controller.

### VI. RESULTS

Fig (5) shows the performance results of PD position controller. Fig (6) shows the performance results of linear PID position controller. Fig (7) shows the performance results of the proposed non linear PID position controller. Fig (8) shows the S.S.E obtained with the PD position controller. Fig (9) shows the S.S.E obtained with the linear PID position controller. Fig (10) shows the S.S.E obtained with the proposed non linear PID position controller.

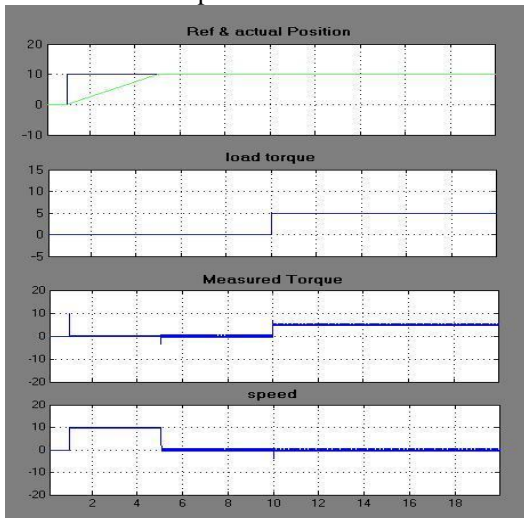


Fig .5 Performance of PD position controller.

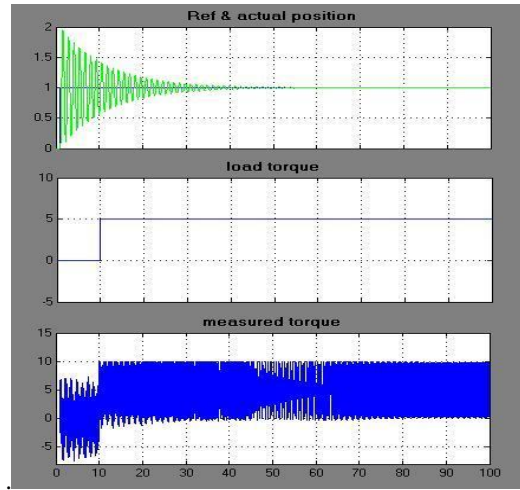


Fig .6 Performance of linear PID position controller.

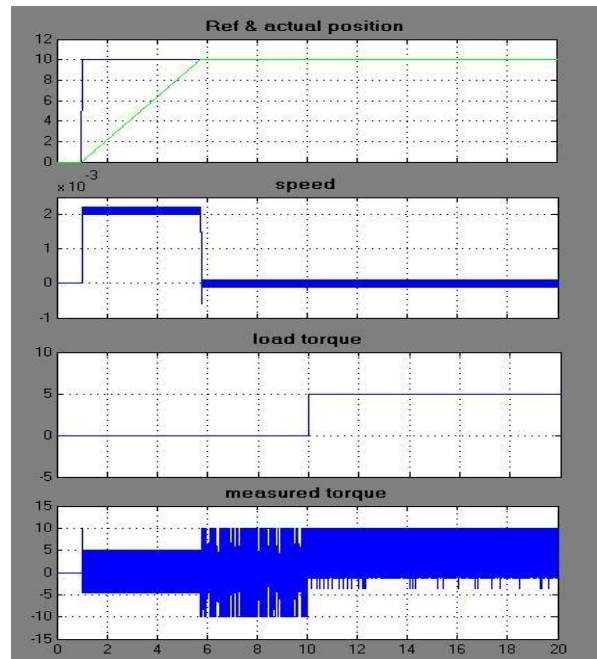


Fig .7 Performance of proposed non-linear PID position controller.

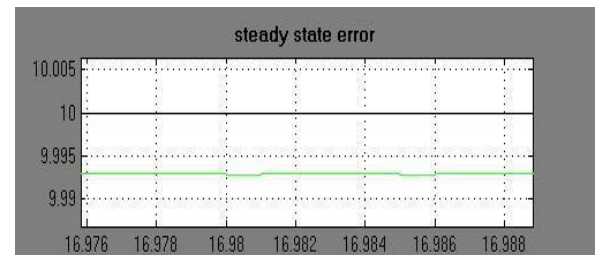


Fig .8 S.S.E obtained with the PD position controller.

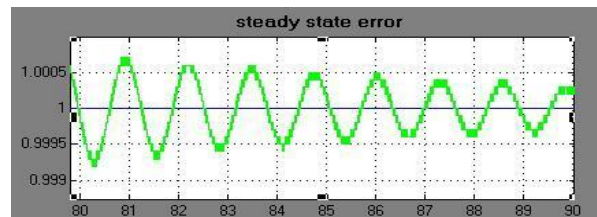
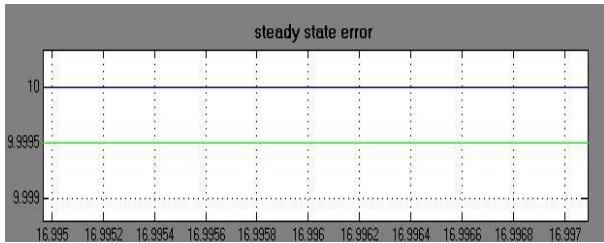


Fig .9 S.S.E obtained with the linear PID position controller.





**Fig .10 S.S.E obtained with the proposed non linear PID position controller.**

## VII. CONCLUSION

The results obtained for non linear PD position controller, given in fig (5) prove that, aperiodic response was obtained for change of reference input position.

With a load torque disturbance of 5 N-m, S.S.E occurred is 0.007 rad as seen in fig (8). This S.S.E value depends on value of load torque disturbance.

For the case of linear PID control, the S.S.E occurred is  $\pm 0.0005$  at 0.85 sec from fig (9). The requirement of aperiodic response is lost. From fig (6) it was observed that the system is stable for only small step changes of the reference position. For a step change of 5 rad the system lost stability. From fig (9), it was observed that the time taken for zero S.S.E could be long.

While from fig (7) & fig (5) it was observed that the rise time for non linear PD controller was 0.04 sec. whereas for non linear PID controller it was 0.0475sec which could not be a serious limitation.

Finally from fig (7) it was observed that the proposed method produces a strict aperiodic response. The major advantage observed from fig (10) was that the S.S.E is 0.0005 rad which is very less compared with 0.007 rad obtained with non linear PD controller.

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