Effect of Diaphragm Geometry and Piezoresistor Dimensions on the Sensitivity of a Piezoresistive Micropressure Sensor using Finite Element Analysis

K.Y.Madhavi, M.Krishna, C.S.Chandrashekhara Murthy

Abstract—The performance of piezoresistive micropressure sensors based on their shape has been studied in this paper. Two sensors based on square and rectangular shaped diaphragms having the same surface area and thickness have been investigated. Performance parameters like the maximum induced stress, deflection and sensitivity of the diaphragms have been compared using the finite element tool ANSYS 10.0. An evaluation of the stress profile across both the diaphragms has been done. The role played by the dimensions of the piezoresistors in determining the performance of the sensor has been analyzed in detail using the computer aided design (CAD) tool Intellisuite. The analysis shows that the square diaphragm based sensor is more sensitive and has a higher gauge factor than the rectangular one but the stress profile of a rectangular based sensor is more suitable for making the placement of the piezoresistors less error prone. It has also been found that the variation in the length of the piezoresistor plays a greater role in determining the sensitivity of the sensor than width and thickness variations. From the results of the simulations the shape and design of the sensor can be optimized for a given pressure range.

Index Terms—***: Diaphragm geometry, Finite element analysis, Micropressure sensors, Piezoresistance.***

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I. INTRODUCTION

Since the discovery of piezoresistivity silicon-based piezoresistive micro pressure sensors have been widely used [1, 2] in a variety of applications in biomedical [3], oceanographic [4], and aeronautical [5] sectors. Micro pressure sensors work on the principle of mechanical bending of thin diaphragms by contact media like gases and fluids. The mechanical deformation thus caused in the diaphragm is commonly sensed by piezoresistive or capacitive methods. Though capacitive sensors have the advantage of greater pressure sensitivity and decreased temperature sensitivity they suffer from non linearity and excessive signal loss from parasitic capacitance [6-7]. Whereas piezoresistive sensors are preferred due to various advantages such as excellent linear input /output relationship, small size, low phase lag, large dynamic range easy integration with electronics and a well matured fabrication process [8-9]. A typical piezoresistive pressure sensor consists of two main components, a diaphragm and four piezoresistors. The piezoresistors are arranged in the form of a Wheatstone's network over the diaphragm to obtain an electrical output. The diaphragm which is the main sensing element can be square circular or rectangular. But rectangular or square ones are commonly used since they occupy lesser area, enable easier lithography and fabrication compared to circular ones [10]. Various researchers have studied the effect of diaphragm geometry and the role played by the piezoresistor size and position in deciding the performance of the sensor [11-14]. However detailed data regarding the stress profile induced across the diaphragm of different geometries and the role played by piezoresistor dimensions on the performance of the pressure sensing diaphragm is not available. This paper investigates the effect of the geometry of the diaphragm and variation in piezoresistor dimensions on the sensitivity of a silicon based piezoresistive micro pressure sensors.

The layout of the paper is as follows in section II the mechanics of the diaphragm structures are studied according to the theory of small scale deflections. Section III describes the finite element analysis (FEA) done using ANSYS 10.0 in which the maximum stress induced, the maximum deflection produced and the stress profile of a rectangular and square diaphragm having the same surface area and thickness have been determined for a given pressure range and compared with the analytical expression for validation. Section IV discusses the results of the piezoresistive analysis done using Intellisuite on both the sensors in which the sensitivities of the sensors are compared and the effect of variation in the dimensions of the piezoresistors on the sensor performance is also evaluated. Conclusions are presented in section V.

II. THEORY OF SMALL DEFLECTIONS

When a uniform pressure P(x,y) acts on a diaphragm normal to its surface as depicted in Fig. 1 the diaphragm undergoes a strain giving rise to

- Normal stresses σx,σy which in turn give rise to bending moments M_x and M_y
- Shear stress τ_{xy} which in turn gives rise to the twisting moment M_{xy}

The governing equation for determining the deflection w (x,y) of a diaphragm with a uniform thickness, and perfectly clamped edges subjected to an applied pressure (P) can be derived from the small scale deflection theory and is given as (1) [15].

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K.Y.Madhavi, Department of Physics, Maharani's Science College for Women, Bangalore-560 001, India

M. Krishna, Research and Development, Department of Mechanical Engineering, R V College of Engineering, Bangalore-560 059, India

C.S Chandrashekhara Murthy, Research and Development, R V College of Engineering,Bangalore-560059, India

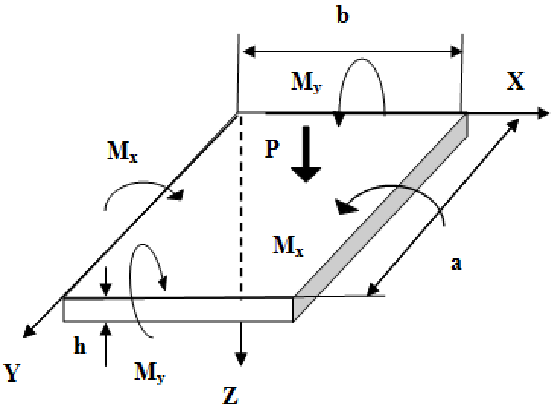

Figure 1: Schematic diagram depicting induced moments in a rectangular plate fixed on all four edges deformed under an applied pressure P.

$$\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial y^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)=\frac{P}{D} \qquad (1)$$

Where D is the flexural rigidity given by

$$D=\frac{Eh^3}{12\left(1-\upsilon^2\right)} \qquad (2)$$

Where E is the Young's modulus ν the Poisson's ratio and h the thickness of the diaphragm.

The solution of Eq. (1) gives the maximum deflection (w_0) at the centre of the diaphragm in the Z axis direction. Having computed w(x,y) the bending moments M_x, M_y and twisting moments M_{xy} per unit length of the diaphragm are denoted as [15]

$$M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \qquad (3)$$

$$M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right) \qquad (4)$$

$$M_{xy}=-D(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right) \qquad (5)$$

The normal stresses σ_x,σ_y and shear stress τ_{xy} can be written as [15]

$$\sigma_x=\frac{6M_x}{h^2}=\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \qquad (6)$$

$$\sigma y=\frac{6My}{h^2}=\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right) \qquad (7)$$

$$\tau_{xy}=\frac{6Mxy}{h^2}=\frac{-2Ez}{2(1-\nu)}\frac{\partial^2 w}{\partial x \partial y} \qquad (8)$$

Where, z is the vertical distance from the diaphragm centre. The solution of equations (6-8) gives the maximum induced stresses (σ_x,σ_y,σ_{xy}) in the diaphragm due to the applied pressure P. Eq. (1) is a complicated partial differential equation and analytical solutions obtained using [16] is described as follows

A. Rectangular Diaphragm

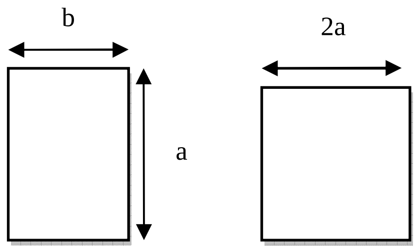

Figure 2: Schematic representation of (i) rectangular diaphragm and (ii) square diaphragm.

For a rectangular diaphragm of length *a* width *b* and thickness *h* as shown in Fig. 2 (i),
The maximum deflection (w_o) and stress (σ_x) for an applied pressure P are given as

$$w_o=\alpha\frac{Pa^4}{Eh^3} \qquad (9)$$

$$\sigma_x=\beta\frac{Pa^2}{h^2} \qquad (10)$$

Where the coefficients α and β are given in table I

Table I: Coefficients α and β for varying a/b ratios [16]

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a/b	1	1.2	1.4	1.6	1.8	2	∞
α	0.0138	0.0188	0.0226	0.0251	0.0267	0.0277	0.0284
β	0.3078	0.3834	0.4356	0.4680	0.4872	0.4974	0.5000

B. Square Diaphragm

For a square diaphragm of side length *2a* and thickness *h* as shown in Fig. 2 (ii)
The maximum deflection (w_o) and stress (σ_x) in the diaphragm for an applied pressure P are given as [16]

$$w_0=\frac{(1-\nu^2)Pa^4}{4.13Eh^3} \qquad (11)$$

$$\sigma_{max}=1.2P\frac{a^2}{h^2} \qquad (12)$$

III. FINITE ELEMENT ANALYSIS

Using the Shell 63 module of the FE tool ANSYS a rectangular diaphragm of dimensions 800 µm x 500 µm and thickness 10 µm and square pressure sensing diaphragm of side length 632 µm and thickness 10 µm have been constructed such that they have the same surface area and thickness. The material properties of silicon used for simulation are given in table II. The maximum stress induced and the deflection produced in the diaphragm are determined and compared with the analytical solutions for a pressure range of 10 kPa – 110 kPa. The results obtained from the FEA are shown in Figs.3 and 4. From the analysis done it can be seen that the maximum stress induced at the edge of the diaphragm and the maximum deflection produced at the centre of the diaphragm are in agreement with the analytical expressions given by equations (9-12). Hence FEA is used for determining the performance of the diaphragm as it is more efficient, time saving and reliable.

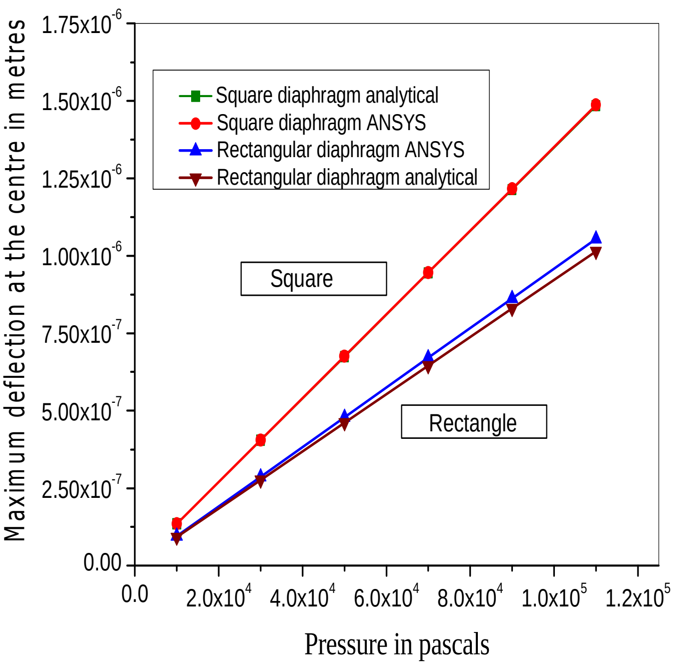
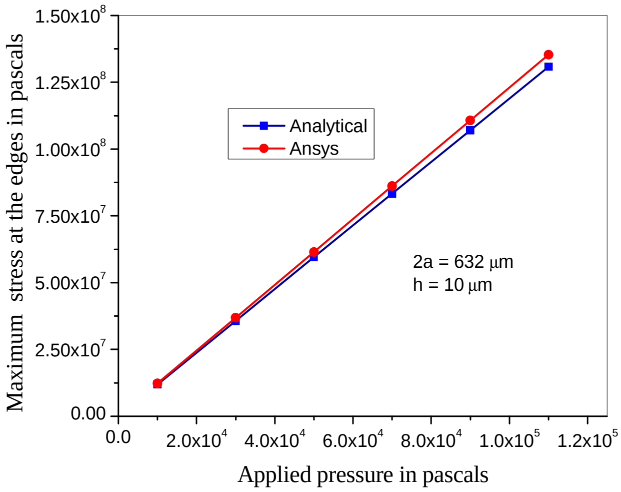


Figure 3: Comparison of ANSYS and analytical values for maximum deflection.

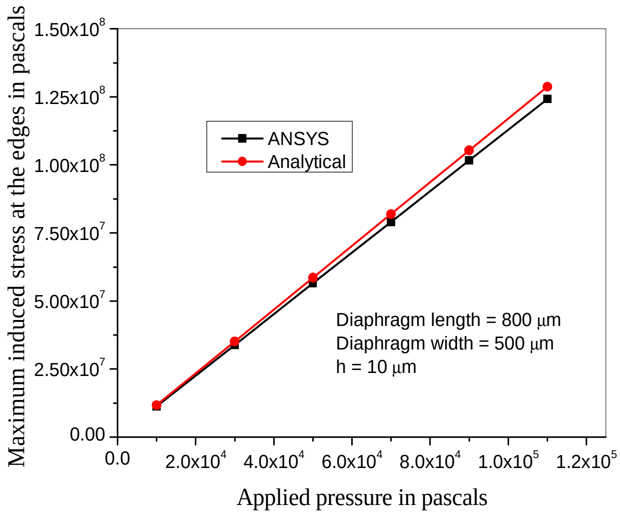
From Fig 3 it can be seen that the maximum deflection of the square diaphragm is 1.4 times that of the rectangular one and from Fig. 4 the maximum stress induced in the square diaphragm is 1.05 times that of the rectangular one thus proving that square diaphragms are more sensitive.

Table II: Properties of silicon used in simulation

Young's Modulus	Density in kg/m^3	Poisson's ratio
170 GPa	2300	0.22



(a)



(b)

Figure 4: Comparison of ANSYS and analytical values for maximum induced stress for (a) Square diaphragm and (b) Rectangular diaphragm.

Having compared the deflection and the maximum stress values of square and rectangular diaphragms the stress profile along X = 0 and Y = 0 axes for a pressure of 100 kPa is studied using ANSYS and the results are depicted in Figs. 5 and 6. From Fig. 5 it can be seen that the stress profile is fairly similar for a square and rectangular diaphragm along Y = 0 axis and that maximum longitudinal tensile stress is experienced at the edges and maximum compressive stress is experienced at the centre. Thus stress concentration areas are at the edges and the centre where the piezoresistors can be placed in order to obtain maximum sensitivity. From the study of the stress profile in Fig.5 it can be seen that compressive stress which is negative is experienced at the centre across a distance of 150 µm for the rectangular diaphragm and across a distance of 380 µm for a square diaphragm. Tensile stress which is positive is experienced up to a distance of 100 µm from the edges for a rectangular diaphragm and up to a distance of 95 µm from the edges for a square diaphragm.

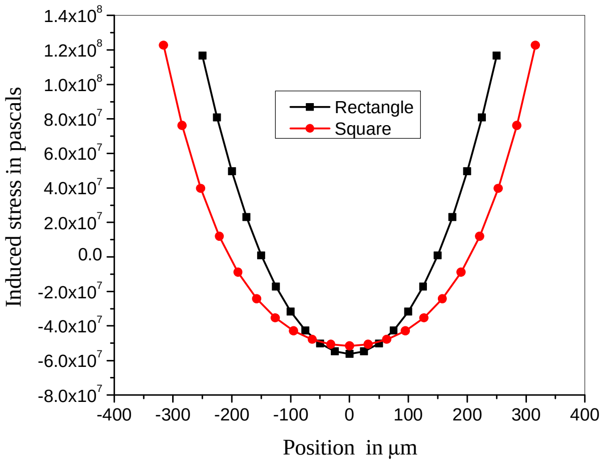


Figure 5: Longitudinal stress profile along X = ±a and Y = 0

From Fig. 6 it can be seen that for a rectangular diaphragm when the stress profile along X = 0 axis is considered the stress distribution is more uniform at the center than the square one thus making the placement of the piezoresistors more easier at the centre of a rectangular diaphragm. It can also be seen from Figs 5 and 6 that the stress profile is symmetrical along X = 0 and Y = 0 axes for a square diaphragm thus enabling equal changes in resistance when a pressure is applied, whereas for the rectangular diaphragm the stress profile along X = 0 axis is different from that along Y = 0 axis

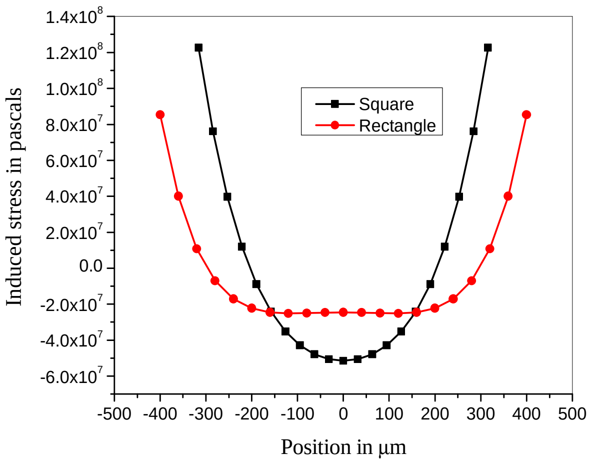


Figure 6: Longitudinal stress profile along Y = ±a/2 and X = 0

IV. PIEZORESISTIVE ANALYSIS

The study of the stress profile enables the strategic placement of piezoresistors in order to obtain maximum sensitivity. From the stress profile in Figs. 5 and 6 it is evident that the maximum stress is induced at the centre of the edges of the diaphragm and the piezoresistors are placed here. In general the piezoresistors are arranged in the form of a Wheatstone's network over the diaphragm to obtain an electrical output as shown in Fig. 7.

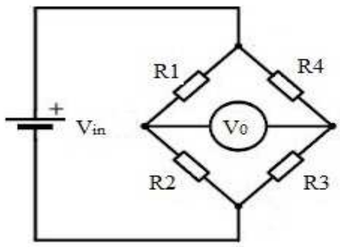


Figure 7: Schematic of the piezoresistors connected in a Wheatstone's bridge

The bridge is balanced under zero pressure condition. When a pressure is applied on the diaphragm all the four resistors undergo a change in resistance and the output of the bridge V_0

$$\frac{V_o}{V_{in}} = \frac{1}{4R_0}(\Delta R1 - \Delta R2 + \Delta R3 - \Delta R4) \quad (13)$$

The gauge factor (G) which is a measure of the piezoresistivity of the piezoresistors is calculated as

$$G = \frac{\Delta R}{R_0 \varepsilon} \quad (14)$$

Where ΔR is the change in resistance for an applied pressure P and R_0 is the initial resistance and ε is the corresponding strain.

The sensitivity of the sensor is calculated using (5) and is expressed as mV/V/bar

$$S = \frac{V_o}{V_{in}} x \frac{1}{\Delta P} \quad (15)$$

Where, V_0 is the output of the sensor for a pressure change of ΔP. Using the FEA tool Intellisuite pressure sensing diaphragms with the patterns of piezoresistors depicted in Fig. 8 have been constructed for both the diaphragms. The masks are designed in the Intellimask module and then auto meshed into the 3-D builder and after assigning the selected dimensions they are finally exported to the thermo-electro-mechanical (TEM) module for piezoresistive analysis and the output of the sensor is determined. The piezoresistive coefficients used in simulation are π_{11} = $6.6x10^{-11}$ Pa^{-1}, $\pi_{12} = 1.1x10^{-11}$ Pa^{-1}, $\pi_{44} = 138.1x10^{-11} Pa^{-1}$[16]. Here initially R_0=R1=R2=R3=R4=1kΩ, and V_{in} = 5V. The piezoresistive analysis was performed on both the sensors for varying dimensions and the parametric performance of both the sensors are compared in Figs. 9-12

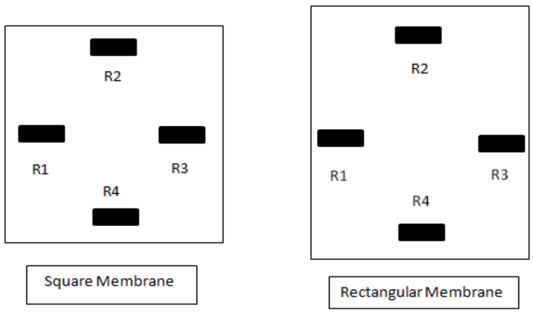


Figure 8: Position of piezoresistors over (a) square diaphragm and (b) rectangular diaphragm.

Fig.9 depicts the variation in the resistance of the piezoresistor as a function of length for three different widths of the piezoresistors. For both the diaphragm geometries the resistors R1 and R3 placed perpendicular to the edge of the diaphragm experience an increase in resistance due to the longitudinal and transverse tensile stresses whereas R2 and R4 placed parallel to the edge of the diaphragm experience a decrease in resistance thus giving rise to a maximum output as seen from (13). From Fig.9 it can be seen that for both the sensors the resistors sensing tensile stress R1 and R3 belonging to group ‘A’ show a greater change in resistance depicted by $\Delta R/R_0$ than the resistors sensing compressive stress namely R2 and R4 represented by group ‘B’. Thus the sensitivity of the sensor is greatly influenced by changes in the resistance of resistors belonging to group ‘A’. It is also observed that the change in the resistance (ΔR) of the piezoresistors decreases with increasing length for both the sensors. Also piezoresistors having a greater width exhibit a smaller change in resistance for both the sensors. These results can be attributed to the fact that as the size of the piezoresistor increases the average induced stress over that area decreases as seen from the stress profiles in Figs 5 and 6 thus leading to a smaller change in resistance and hence to a decrease in output.

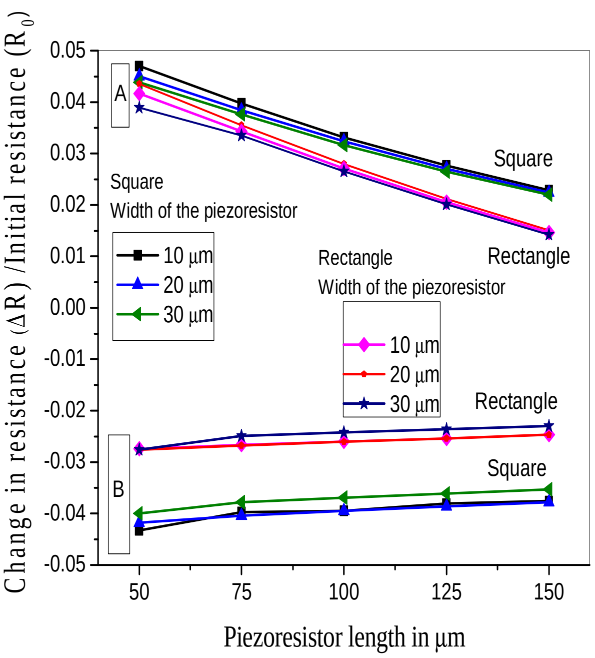


Figure 9: Variation in the resistance of the piezoresistors as a function of piezoresistor length and width for P = 100kPa.

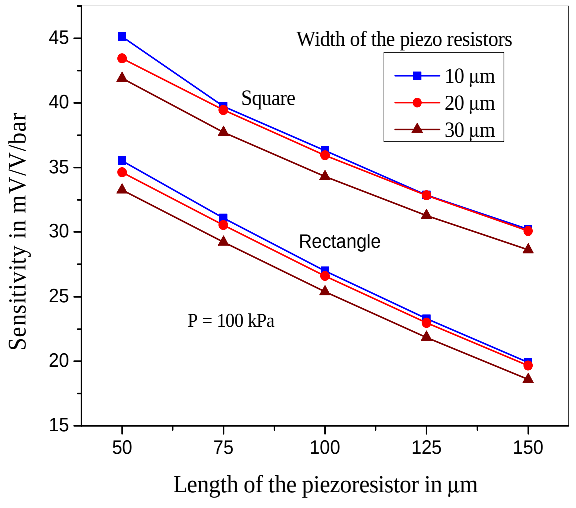


Figure 10: Sensitivity of the square and rectangular based sensor for varying piezoresistor length and width.

Fig. 11 depicts the performance of both the sensors for varying thickness of the piezoresistors. It can be seen that for both the sensors the sensitivity of the sensor decreases as the thickness of the piezoresistors increases. This is due to the fact that there is a decrease in the average induced stress across the resistor as the thickness of the resistor is increased. The square diaphragm based sensor is 1.3 times more sensitive the rectangular based sensor and this ratio is constant over the entire thickness range.

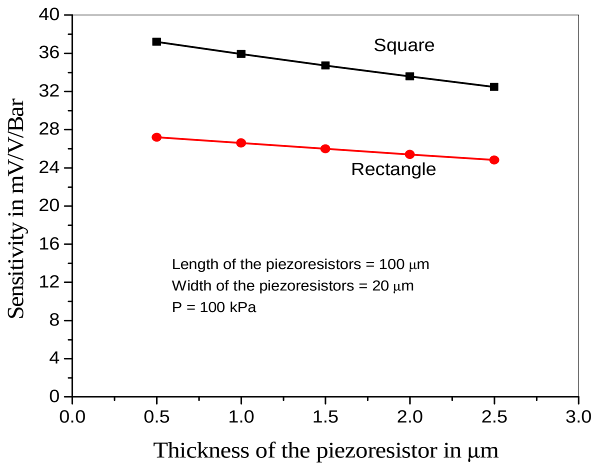


Figure 11: Sensitivity of the square and rectangular based sensor as a function of piezoresistor thickness

Comparing the sensitivities of the two sensors it can be observed from Fig. 10 that the sensitivity of the sensor with a rectangular diaphragm is lesser than that of the square one. This is because the maximum deflection produced and the maximum stress induced is lesser for a rectangular diaphragm thus leading to a decrease in sensitivity and gauge factor also. The sensitivity of the square diaphragm is 1.2 times greater than the rectangular one for a piezoresistor length of 50 µm and this ratio increases to 1.5 for a length of 150 µm.

Fig 12 gives us a comparison of the gauge factors of the piezoresistors placed on both the sensors calculated using (14). From Fig. 12 It can be seen that the gauge factor of the square diaphragm based sensor is higher than that of the rectangular based one even though the piezoresistors are of the same dimensions. It is also observed that the gauge factor of both the sensors decreases with increasing length of piezoresistors thus proving that smaller resistors are more sensitive as the average induced stress across them is higher than that of a larger resistor.

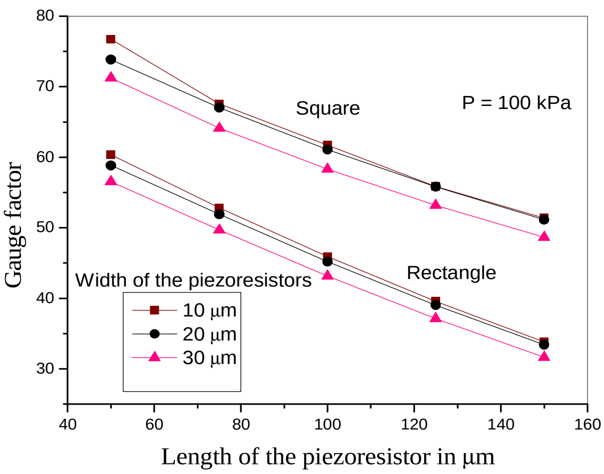


Figure 12: Gauge factor of the square and rectangular based sensors as a function of piezoresistor length and width.

V. CONCLUSION

In this paper the effect of diaphragm geometry and piezoresistor dimensions on the sensitivity of a piezoresistive micropressure sensor have been analyzed using finite element tools ANSYS and Intellisuite. The investigation clearly indicates the outcome of using different geometries for the diaphragm and enables the estimation of piezoresistor dimensions. From the analysis done the followings results have been obtained

- The square diaphragm has a greater deflection and a slightly higher induced stress for a given pressure compared to a rectangular one and hence it is more sensitive and has a higher gauge factor.
- The induced stress remains more uniform at the centre of a rectangular diaphragm than a square one hence making the placement of the piezoresistors in that area less error prone during fabrication.
- The piezoresistors R1 and R2 sensing tensile stress show a greater variation in resistance when their dimensions are altered than that of resistors R2 and R4 sensing compressive stress. Hence resistors R1 and R2 play a crucial role in determining the sensitivity of the sensor.
- The variation in the length of the piezoresistor plays a greater role in determining the sensitivity of the sensor than width and thickness variations. In general piezoresistors of smaller dimensions are more sensitive.

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K.Y.Madhavi, has obtained her M.Sc and M.Phil degree in Physics from Madras Christian College affiliated to the University of Madras, India and is currently working as Assistant Professor at Maharani's Science for Women. Bangalore.. Her main area of interest involves MEMS and thin films.

Dr. M Krishna BE, MS, has acquired his Ph.D. in Materials Science from Mangalore University, India. He is working in the area of Composites, Alloys, Corrosion & electroplating. He has published more than 100 research papers in international refereed journals. He is currently the Director of CMRTU (R&D) and Professor of Dept. of Mechanical Engg, R V College of Engg, Bangalore, India.

Dr. C.S.Chandrashekhara Murthy, has obtained his BE degree in Mechanical Engineering from the University of Mysore, India and has a Ph.D from Indian Institute of Technology Madras He has over 39 years of teaching experience in R V College of Engg, Bangalore, India. He has published more than 20 research papers in international and national journals. His main areas of research interest are Materials Science and Solid Mechanics and Design.