Design and Implementation of Robust Digital Redesigned Controller to Balance an Inverted Pendulum System

Kanthalakshmi. S, Vivekandan. C, Kavithamani. A, Manikandan. V

***Abstract*— This paper aims at designing a robust digital redesigned controller for a system of inverted pendulum. The issues considered for evaluation of the designed controller are the ‘closeness’ between the closed loop response of the continuous-time and discrete-time system and the stability of the redesigned digital system. The closeness aspect between the continuous-time system and its discrete-time equivalent is measured in the form of the integral error performance index and the stability of the redesigned system is ascertained in the sense of Lyapunov. The error in the digital redesign process is reduced using Feed Forward Back Propagation Neural Network Approach. The robustness and stability are achieved and tested with Lyapunov criteria. The design is practically verified with a real time implementation.**

***Index Terms*— Digital Controller, Digital Redesign, Neural Networks, Robustness, Lyapunov Stability.**

I. INTRODUCTION

Most complex dynamical systems, including chaotic systems, are described by continuous - time models. It is, therefore, common practice and in fact, advantageous to design a controller in the continuous - time framework meeting specific design goals. The continuous-time controller, on the contrary, is preferable to be implemented through a digital device for better performance, greater flexibility and lower cost. A digital implementation of the continuous - time controller is indeed highly desirable if the designed continuous-time control uses advanced control techniques like Sliding mode control [Utkin. V. I., 1977], Quantitative Feedback Theory [John J. D’Azzo, Constantine H. Houpis, 1988] etc., However, many digital controller attempts to approximate the continuous - time controller with a discrete-time equivalent assuming that the sampling period is sufficiently small. As a consequence of this approximation, the resulting discrete - time controlled system may become unstable even if the original continuous - time controlled system is stable. Even if the discrete - time controlled system is stable, the states of the discrete - time system may not closely match with those of the original continuous - time control system leading to the complete failure of the design process.

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Hence perfect matching between the states of the analog continuous - time system and its digital equivalent with guaranteed closed loop stability are not only vital but contradicting issues as well that needs to be addressed at length while designing a digital controller for a continuous-time plant.

The conventional approach for designing a digital controller for a continuous-time system is to first discretize the analog system and then design a controller directly in the digital domain to meet control specifications that are suitably transformed from continuous - time domain (s-domain) to discrete time (z-domain) using mapping techniques like e^{sT} transformation, bilinear transformation etc. Of course the direct digital design based on these mapping techniques work satisfactorily only for sampling times that are considerably small and well within the bounds furnished by Shannon’s sampling theorem.

Another technique quite popular amongst the control community in the recent past is the LMI based digital redesign technique [Chang. W., et. al. , 2002, Lee. H.J., et. al. 2003, Lee. H.J., et. al., 2006]. A comparative evaluation of various approaches is done in [Rabbath. C.A., Hori. N., 2000, Sivanandakumar.D., 2007] . The digital redesign procedure converts a satisfactorily designed analog controller into an equivalent digital controller such that the states of the redesigned digital system match those of the original analog system for the same reference input and initial conditions.

The paper is organised as follows: Digital redesign concepts and various digital redesign techniques are discussed in sections II and III. The Neural Network approach to reduce the error between the analog controller response and digital counterpart is discussed in section IV. Results and discussion are presented in section V and section VI gives the concluding remarks.

II. DIGITAL REDESIGN

Consider the Linear Time - Invariant continuous - time system [8]

$\dot{X}_C(t) = AX_C(t) + Bu_C(t),\ X_C(0) = X_0$ → (1)

$y_C(t) = CX_C(t)$

where $X_C(t) \in \Re^n$ is the state vector, $u_C(t) \in \Re^m$ is the control vector, $y_C(t) \in \Re^p$ is the Output vector; A, B and C are constant matrices of appropriate dimension. The control vector, $u_c(t)$ is given by[8]

$u_C(t) = -K_C X_C(t) + E_C\, r(t)$ →(2)

where $K_C \in \Re^{mxn}$ and $E_C \in \Re^{mxn}$ are state feedback gain and feed – forward gain respectively; $r(t) \in \Re^p$ is the reference input. The closed loop system shown in Fig.1 is given by [8]

$dx_C/dt = (A - BK_C)\, X_C(t) + BE_C\, r(t),\ X_C(0)=X_0$ →(3)

$y_C(t) = CX_C(t)$

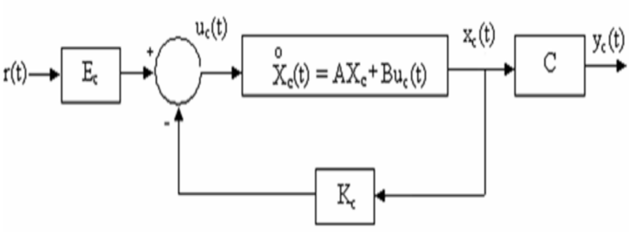

Fig.1 Closed Loop System

The discrete model of the closed loop system is given by

$X_C(kT+T) = \hat{G}X_C(kT) + \hat{H}E_C\, r(kT)$ →(4)

where,

$\hat{G} = e^{(A-BK_c)T}$ and $\hat{H} = \int_0^T e^{(A-BK_c)\lambda}\, Bd\lambda$; $k \in Z^+$ and T is a non - pathological sampling time.

Consider the continuous - time system in Equation (1) with digital control input, $u_d(t)$ as shown in Fig. 2

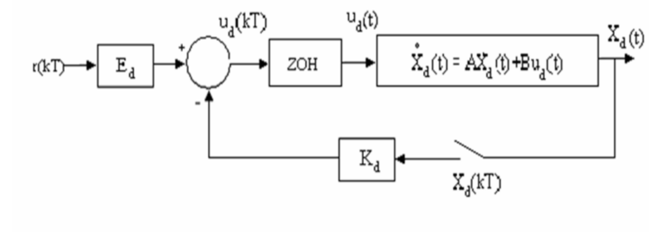

Fig.2 Closed loop system with digital control input

The closed loop system equation is given by

$X_d\,(kT+T) = (G-HK_d)\, X_d\,(kT) + HE_d\, r\,(kT)$ →(5)

$y_d\,(kT) = CX_d\,(kT)$

where $K_d \in \Re^{mxn}$ and $E_d \in \Re^{mxp}$ are the digital equivalents for gains K_C and E_C respectively and

$G = e^{AT}$ and $H = \int_0^T e^{A\lambda}\, Bd\lambda$ →(6)

The objective of the redesign problem is to compute the digital gains, K_d and E_d such that the states of the discrete-time system, $X_d(kT)$ match with those of the continuous-time system, $X_C(kT)$ as closely as possible at all sampling instants.

III. DIGITAL REDESIGN TECHNIQUES

The existing methods of digital redesign are:

- Taylor Series Approximation Method
- Bilinear Transformation Method
- Block Pulse Approximation Method
- Linear Matrix Inequality (LMI) Based Method

The digital gains K_d and E_d using Taylor series approximation is given by,

$K_d = K_c + 1/2\, K_c(A - BK_c)T$

$E_d = (I_m - ½\, K_cBT)\, E_c$

The digital gains using Bilinear Transformation is given by,

$K_d = K_c(I_n - ½(A - BK_c)T)^{-1}$

$E_d = (I_m - ½K_dBT)E_c$

In Block Pulse Approximation, the control input is expanded in block pulse series described by,

$U_c = \Sigma\, C_i\varphi_i(t)$

where $\varphi_i(t) =$ 1 for $iT<t<(i+1)T$

0 otherwise

and weighting constants C_i are evaluated from

$C_i = 1/T \int_{iT}^{(i+1)T} u_c(t)\varphi_i\,(t)dt$

The following two constraints are incorporated as two LMI constraints in Generalised Eigen Value Problem (GEVP)

$(G_c - (G-HK_d))^T(G - HK_d) < \alpha^2P$

$(G-HK_d)^TP(G - HK_d) - P < 0$ (Lyapunov)

where $P = P^T > 0$

The GEVP is solved and digital gain K_d is obtained. The forward gain E_d is computed using shifted regulator approach.

IV. ERROR REDUCTION USING ARTIFICIAL NEURAL NETWORKS

In order to reduce the error between the analog controller response and digital counterpart, the Feed Forward Back Propagation Network (FF-BPN) can be used. During the training phase the set of input – output vectors are used to update the weight matrices V and W to minimize the error function. The total error between the desired and actual state variables is then back propagated so as to adjust the weights of the neural model such that states and output of the neural model coincides with the states and output of the continuous-time model.

The block diagram of the proposed approach is shown in Fig. 3.

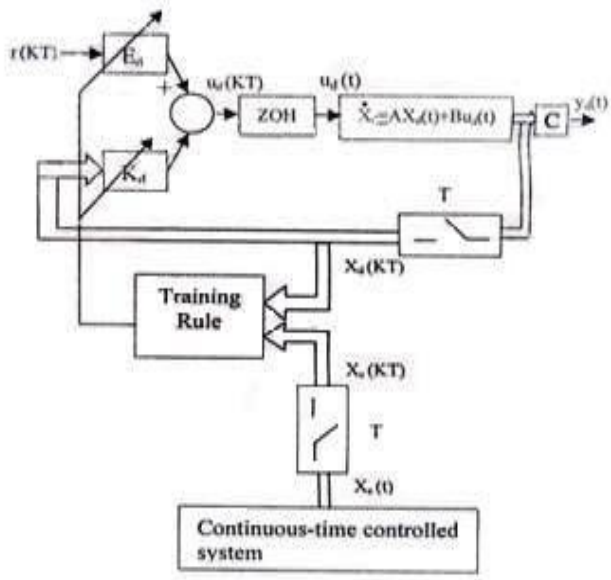

Fig.3 The block diagram of the ANN included digital redesigned controller

The stability of the Robust Controller is tested using the Lyapunov Stability Criteria and the inequalities are satisfied.

Lyapunov Stability: For all matrices $Q = Q^T > 0$ there exists an unique solution $P = P^T > 0$ to the following (Lyapunov) equation [8]: $A^T P + PA = -Q$

Here, by taking $Q = I_n = I_{4x4}$ is taken and solved for P. Then it is used to test the Stability of the system using Lyapunov Stability Criteria given by: $(G - HK_d)^T\, P\, (G - HK_d) - P < 0$.

V. RESULTS AND DISCUSSION

Consider the unstable inverted pendulum system given by,

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & -17.25 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & -53.9 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ -1 \end{bmatrix} U$$

$Y = [\,1 \quad 0 \quad 0 \quad 0\,]\, X$

The Wolfram Mathematica8 simulation results for inverted pendulum system are shown below. The unstable inverted pendulum is made stable with the suitable analog controller as shown in Fig. 3.

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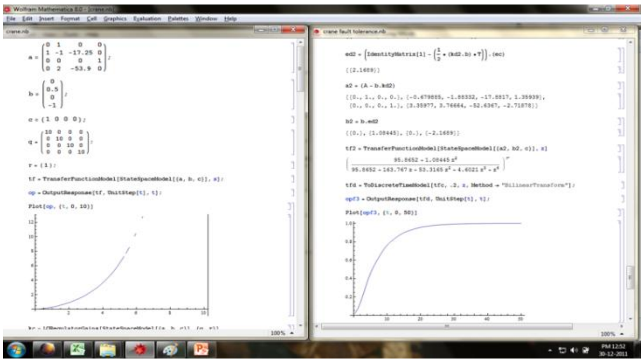

Fig.4 Unstable inverted pendulum system made stable using analog controller

The continuous time controller for the position control of the inverted pendulum system is designed based on LQR approach and the controller gains are given below.

K_c = [3.1623 2.8864 -14.9723 -4.3837]

E_c = [3.1623].

The performances of digital redesigned controller with various techniques for different sampling period are compared with the analog controller and the integral error index is given in Table 1.

Table1: Comparison of Integral Error Index using various redesign techniques

INTEGRAL ERROR INDEX			
METHOD	T = 0.2	T = 0.5	T = 0.6
Taylor Series Approximation	0.6980	Unstable	Unstable
Bilinear/Block Pulse Transformation	0.0324	Unstable	Unstable
LMI without Lyapunov Stability Constraint	0.2074	0.2967	Unstable
LMI with Lyapunov Stability Constraint	0.2312	0.1406	0.4440

The ANN used for the simulation and training of the inverted pendulum system is Feed Forward Back Propagation Network (FF – BPN) with linear sinusoidal signal as the activation function and the order of ANN is 2:2:1

The error in the redesigned controller reduces significantly with the inclusion of ANN. For example, the error in Taylor series Approximation method reduces from 0.6980 to 0.0182 with the Percentage Error Reduction (PER) of 97.39% (shown in Fig. 5).

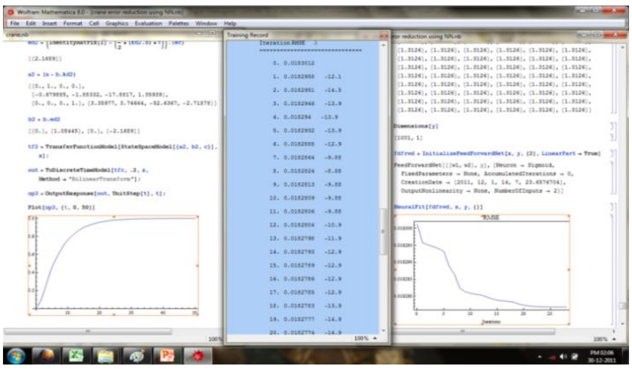

Fig.5 RMSE values and graph with respect to the number of iterations of training of the FF-BPN.

The external fault is simulated into the system and the behaviour of the controller to accommodate the fault is verified. The controller is found to be robust enough to withstand the faulty condition as shown in Fig. 6 and Fig. 7

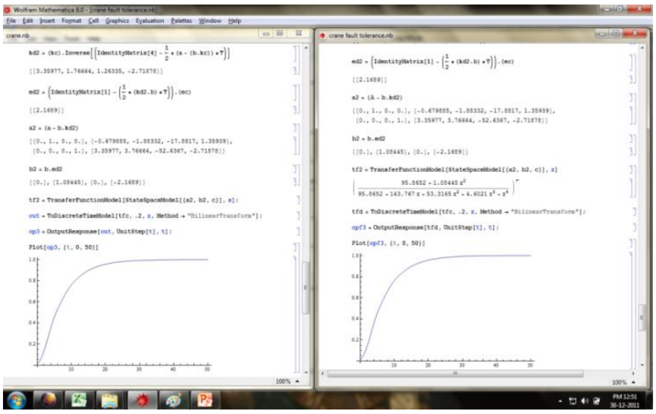

Fig.6 System response without fault (left) and with fault (right) -- Fault Tolerance

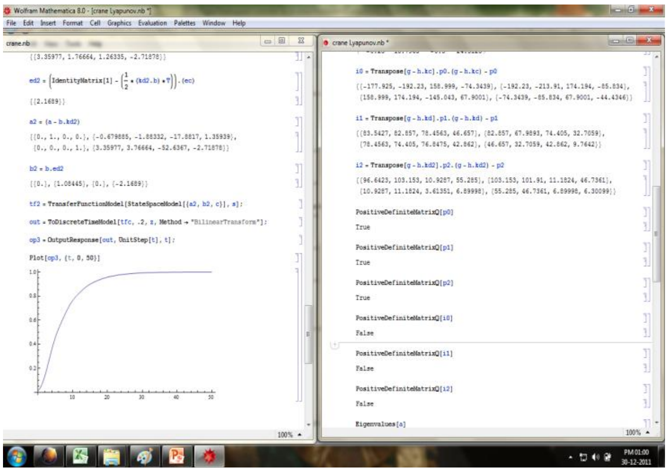

Fig.7 Screenshot showing the satisfaction of the Lyapunov Stability Criterion using Wolfram Mathematica 8.

The hardware implementation of inverted pendulum setup consists of three main parts: the base platform, the pendulum and the controller board as shown in Fig. 8. The base platform is a 3 point platform with 2 wheels (one of which is geared and attached to a DC motor) and an audio jack. When the DC motor is turned on, the base platform will rotate around in a circle with the centre of the axis of rotation being the audio jack. In addition, audio jack is also used to bring the commutated power to the controller board.

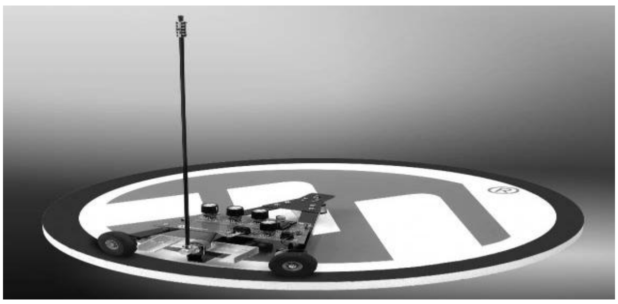

Fig. 8 Inverted Pendulum setup

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The pendulum is attached to the base platform by a 360^o free rotating potentiometer. The pendulum's base is attached to a potentiometer in such a fashion that when the pendulum is balanced(completely vertical),the potentiometer centre tap is biased to $V_{ref}/2$.

The controller board has two main functions: to measure the displacement angle of the pendulum with respect to the vertical axis and to drive the DC motor. The power supply needed to run the system is dictated by the selection of the motor. The motor is controlled by the H bridge which is driven by the PIC16F684. There are five potentiometers located on the controller board: three of which are used to adjust the controller gains, one the measure the displacement angle and the fifth one to control the input filter's reference to produce a true 0^o displacement angle when the pendulum is vertical.

The inverted pendulum system is given an external fault by distorting the stable pendulum. Due to this distortion, the angular potentiometer on which the pendulum is mounted will change the voltage across it and thereby sends the instruction to the controller to change the direction and speed of the motor such that the pendulum will stand erect again. The controller brings back the pendulum to the vertical position as long as the external disturbance causes the distortion less than 10^o. If the fault limit exceeds, the controller is currently unable to get back the pendulum to its stable position and thus results in system failure.

V.CONCLUSION

All the existing methods for digital redesign are studied and tested for an unstable inverted pendulum system and the error present in those methods is reduced using the concepts of Artificial Neural Networks, irrespective of the method used and the sampling period taken. The controller designed is a robust and fault tolerant controller. Also the stability of the system is checked using the Lyapunov Stability Criterion. The hardware implementation also proves the efficiency of the proposed method by providing fast and accurate results with significant reduction in percentage error.

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