Statistical Methods Application for Estimation of Unit Costs in a Cogeneration Plant Sugar Industry

Vindhya, Sunil Kumar, Madhuraj

Abstract—India is currently largest producer of Cane-Sugar in the world accounting for 10% of the world production. Most Cane –Sugar factories have been designed to be energy self-sufficient with sugar as the Primary product. With the recent trend toward diversification in the Cane-Sugar Industry, a growing no. of factories are manufacturing one or more byproducts (such as alcohol or cogenerated electricity for export to the utility grid) in addition to Sugar and Mollases. Co-generation is the concept of producing two forms of energy from one fuel. One of the forms of energy must always be the heat and the other may be electricity or mechanical energy. A method for establishing unit costs of delivered steam and electrical energy is presented. This method employs the use of least squares, based on a linear model of electrical energy generation and delivered steam as functions of generated boiler steam. A discussion of the accuracy of the method is presented as well as an example of the use of the method using four months of actual plant production. An excel program is discussed for solving the solution of simultaneous equation generated after least squares approximations. Accordingly comparision among all the adopted methods are carried out. This helps in betterapproximation for evaluation of per unit cost for a cogeneration system , which is further used for determining the correct pay-back period of the plant.

Index Terms—Cogeneration, costs, least squares methods, steam generation, steam turbines, surface fitting.

I. INTRODUCTION

Co-GENERATION plants are extremely beneficial and cost effective for large institutions which require both heating and electrical power. This is particularly so when heating and electrical demands are well balanced and the demand for extracted steam and electrical power complement one another closely. Whenever a plant has the optional capability of discharging steam from the turbines either at service pressure or at a vaccum, however, there is a definite unit value which can be assigned to both the electrical energy generated and the service steam delivered. A mathematical model can be developed for cost as a function of both steam and electricity delivered, and the model can be fit to data from the boiler logs by the method of least squares. This provides a systematic method by which unit costs can be accurately calculated. The accurate calculation of unit costs for utilities generated from a plant is very important whenever consumption is metered and billed to differing accounts within an organization. These calculations are also very important for estimation of the payback period of the plant.

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TABLE I. BOILER TURBINE CAPACITIES

Configuration of the Existing Unit	
Boilers	**Turbines**
80TPH, 87 Kg/cm2 Temp.- 515 ± 5^0 C Type- Water Tube	15 MW Voltage Generated-11KV

II. ASHOK S.S.K.L. SRIRAMPUR PLANT SPECIFICS

The 15 MW capacity cogeneration project at M/s. Ashok Sahakari Sakhar Karkhana Ltd (ASSKL) will integrate existing sugar mill operations with enhanced energy efficiency measures and optimum usage of bagasse. During season, generated mill bagasse will be transferred to the cogeneration plant which is to be installed in the campus of existing plant located at Ashoknagar village in (M.S.) India. The cogeneration plant will supply the heat and power requirements of the sugar mill and evacuate excess power to the state owned grid at Babhaleshwar (Ahmednagar). Power is stepped down to 440 V for supplying to sugar mill and cogeneration auxiliaries. Whereas for export to the grid, it is stepped up to 132 KV. For case study of Ashok plant in season 2013-14 data has been taken in account which is listed below Table II. All Steam generated in the plant is eventually condensed and returned to the boilers as feed water. Table II shows the monthly values of the steam delivered to the campus as well as the overall production of boiler steam. The value given for boiler steam includes the steam used in the production of electricity that is condensed at a vaccum, as well as the steam discharged at 87 kg/cm2 and sent to the campus distribution system. For example, during the first month of Oct. 2013 shown in Table II, 35345 MT of Boiler steam was generated and 28510 MT of steam was extracted from the steam turbine and delivered to the campus at 87 kg/cm2 for heating purposes. The remainder was condensed as a vaccum, used exclusively in the electrical generation process. On the annual averaged basis, approximately 53% of boiler steam is extracted from the turbine at 87 kg/cm2 and sent to the campus distribution system. This leaves 47% which is used in the conventional generation of electricity only.

TABLE II. PLANT OUTPUT OF SEASON 2013-14 (04 MONTH)

Month	Delivered Steam{S_d} MT	Electrical Energy(E) KWH	Boiler Steam(S_b) MT
Oct. 2013	28510	5853200	35345
Nov. 2013	28300	5685000	34510
Dec. 2013	26500	5716510	33120
Jan. 2014	27165	5451320	33600

III. MODELING THE PLANT OUTPUT

In order to establish unit cost for the electrical and steam utilities, a mathematical model must be developed which accounts for the fuel consumed in terms of the utilities delivered. We know that there is a certain cost associated with operating the plant even if no utilities are generated whatsoever. The cost of salaries for the staff to operate and maintain the plant, the cost of service contracts for specialized maintenance, and any amortization costs associated with the original construction of the plant are lumped into the categories of "fixed costs."

Even though the fixed cost cannot be easily converted to unit costs for electrical and steam energy delivered, it is desirable to regroup these costs by charging customer's unit costs for the boiler steam generated and then attributed to electrical and steam unit costs from there. The overall boiler steam unit cost can be calculated by the sum of the overall plant costs per year divided by the total no of MT of boiler steam generated.

Expressed in equation form

$$C_{bs} = \frac{A_{maint} + A_{op} + A_{fuel} + \ldots\ldots + A_{cont}}{S_a} \quad \ldots(1)$$

where

C_{bs}	unit cost of boiler steam.
A_{maint}	annual cost of plant maintenance staff.
A_{op}	annual cost of plant operating staff.
A_{fuel}	annual cost of fuel consumed by the plant.
A_{cont}	annual cost of contracted supplies and services.
S_a	annual amount of boiler steam generated.

With a unit cost of boiler steam obtained, the unit costs of delivered steam and delivered electrical energy can then be calculated. In order to do this, the amount of boiler steam attributable to each of the two delivered utilities must be calculated. The mathematical model for making this conversion is as follows:

$$S_b = I + X_s S_d + X_e E \quad \ldots(2)$$

Where

S_b	boiler steam required.
I	internal steam usages.
X_s	delivered steam ratio (boiler steam per delivered steam).
S_d	delivered steam
X_e	electrical steam ratio (boiler steam per electrical steam).
E	delivered electricity in KWH.

The known factors in this equation are Sb, Sd, and E. These are all obtainable from the monthly boiler logs. The time intervals typically used for this equation are of one month duration, since this provides a diverse range of operating conditions to average out any errors or anomalies in the records. These type of irregularities tend to have a more imbalanced effect when measured over shorter periods. Regardless of the time period used, it is important to be consistent in using the same time period for each term in the equation. The reason for this is that the parameter, internal steam usage, varies depending on the time period used; the others do not.

IV. FITTING THE DATA

With the mathematical model developed for the plant output as a function on boiler steam, the data can now be fitted to the model by least squares. For the Sonai plant, data from 4 months of plant operation were used in arriving at the actual estimated parameters. A typical 4 month period is shown in Table II. Substituting the data from Table II into the mathematical model (2) leaves an over-defined set of equations, with the desired parameters as unknowns. In this example, we have 7 equations and three unknowns. This set of equations is partially shown in expanded form as,

35345 = I + 28510 Xs + 5853200 Xe
34510 = I + 28300 Xs + 5685000 Xe
33120 = I + 26500 Xs + 5716510 Xe
33600 = I + 27165 Xs + 5451320 Xe
…(3)

This set of equations can also be more conveniently expressed in matrix form as,

$$Sb = Dp \quad \ldots(4)$$

Where 'Sb' is the vector of monthly boiler steam readings. The dimensions of the Sb matrix in this example are [4x1]. The D matrix represents the other data collected from the boiler logs arranged where the first column is the coefficient if the internal steam usage parameter I. The dimensions of this matrix are [4x3]. Finally, the parameter vector P carries dimensions [3x1]. The objective of this procedure is to find values for the parameters I, Xs, and Xe such that, when substituted into (4), the calculated values for boiler steam most closely match the measured values. Specifically, we desire to minimize the sum of squares of the errors between calculated and measured values of boiler steam Sb. This sum of squares can be computed as follows:

$$S_b = \sum_{i=1}^{n} (S_{b(measured)} - S_{b(calculated)})^2 \quad \ldots(5)$$

In order to find the values for the parameters I, Xs, and Xe such that S is minimized, we simply premultiply each side of (4) by the matrix D which results in

$$D^T S_b = D^T D P \quad \ldots(6)$$

Since the dimensions of the $D^T D$ matrix are [3x3], both sides of this equation are of dimensions [3x1] so that the equations are no longer over defined. Moreover, the only unknown in the equation, the P vector, contains the three desired parameters I, Xs, and Xe such that S is minimized. Equation (6) and its development are given in both (1)and (2).

V. METHODS RESULTS

Multiple Regression Least Square Method

Multiple regression estimates the outcomes (dependent variables) which may be affected by more than one control parameter (independent variables) or there may be more than one control parameter being changed at the same time. An example is the two independent variables x and y and one dependent variable z in the linear relationship case:

z=a + bx + cy

For a given data set(x1,y1,z1), (x2,y2,z2), ..., (xn,yn,zn), where $n \geq 3$, the best fitting curve f(x) has the least square error, i.e.,

$$\Pi = \sum_{i=1}^{n}[z_i - f(x_i, y_i)]^2 = \sum_{i=1}^{n}[z_i - (a + bx_i + cy_i)]^2 = \min.$$

Please note that a, b and c are unknown coefficients while all x_i, y_i, and z_i are given. To obtain the least square error, the unknown coefficients a, b, and c must yield zero.

First derivative.

$$\frac{\partial \Pi}{\partial a} = 2\sum_{i=1}^{n}[z_i - (a + bx_i + cy_i)] = 0$$

$$\frac{\partial \Pi}{\partial b} = 2\sum_{i=1}^{n}x_i[z_i - (a + bx_i + cy_i)] = 0$$

$$\frac{\partial \Pi}{\partial c} = 2\sum_{i=1}^{n}y_i[z_i - (a + bx_i + cy_i)] = 0$$

Expanding the above equations, we have,

$$\sum_{i=1}^{n}z_i = a\sum_{i=1}^{n}1 + b\sum_{i=1}^{n}x_i + c\sum_{i=1}^{n}y_i$$

$$\sum_{i=1}^{n}x_i z_i = a\sum_{i=1}^{n}x_i + b\sum_{i=1}^{n}x_i^2 + c\sum_{i=1}^{n}x_i y_i$$

$$\sum_{i=1}^{n}y_i z_i = a\sum_{i=1}^{n}y_i + b\sum_{i=1}^{n}x_i y_i + c\sum_{i=1}^{n}y_i^2$$

The unknown coefficients a, b, and c can hence be obtained by solving the above linear equations.

Multiple Regression Method used for calculation.

Equations using multiple regression method are:

$\sum S_b = n\ I + X_s \sum S_d + X_e \sum E$
$\sum S_d \cdot S_b = I \sum S_d + X_s \sum S_d 2 + X_e \sum S_d E$
$\sum E \cdot S_b = I \sum E + X_s \sum S_d \cdot E + X_e \sum E 2$
where n= number of months= 04

Employing the least squares method on the sample data given in Table II, the resulting parameters are:
136575 = 4 I+110475 Xs +22706030 Xe

3774742950=110475I+3053897325 Xs + 6.273328*10^11 Xe

7.755658672*10^11 = 22706030 I + 6.273328548*10^11 Xs +12.89745516*10^13 Xe

I) By using the values of above chart in multiple regression equations we get,
I= 2566.273559
Xs= 0.903995465
Xe=1.164492726*10^-3 .

II) **By Using Excel** : we can also evaluate the same above result by using Excel as-

Let we have to find the solution of linear algebraic equation of the form [A]{X}={B}.
So,
{X} = [A]$^{-1}$ {B}.

Excel has built- in function for both matrix inversion and multiplication that can be used to implement this formula. Here first, the matrix [A] and right-hand-side constants {B} are entered into the spreadsheet cells. Then, a set of cells of the proper dimensions (in our example 3*3) is highlighted by either clicking or dragging the mouse or by using the arrow keys while depressing the shift key. As in figure we highlight the range: B5..D7. Next, a formula involving the matrix inverse function is entered,
=minverse(B1..D3)

Note that the argument is the range holding the elements of [A]. The *Ctrl* and *Shift* keys are held down while the *Enter* key is depressed. The resulting inverse of [A] will be calculated by Excel and displayed in the range B5..D7.
A similar approach is used to multiply the inverse by the right-hand-side vector. For this case, the range from F5..F7 is highlighted and the following formula is entered
=mmult(B5..D7, F1..F3)

Where the first range is the first matrix to be manipulated,[A]$^{-1}$, and the second range is the second matrix to be multiplied,{B}. By again using the *Ctrl-shift-Enter* combination, the solution {X} will be calculated by Excel and displayed in the range F5.F7.As can be seen, the correct answer result.

So by using this the result is obtained as-

TABLE – III. RESULT AFTER EXCEL COMMAND

	4	110475	22706030		136575
[A]=	110475	3.05E+09	6.27E+11	{B}=	3.77E+09
	22706030	6.27E+11	1.29E+14		7.76E+11
	460.5023	-5.92E-03	-5.23E-05		2566.28
[A]-1=	-5.92E-03	4.69E-07	-1.24E-09	{x}=	0.904
	-5.2E-05	-1.24E-09	1.52E-11		0.00116

so,
I=a= 2566.28154
Xs=b= 0.90399571
Xe=c= 0.00116449

III) **By Using Mat Lab**:-

If *y* is a function of more than one independent variable, the matrix equations that express the relationships among the variables are expanded to accommodate the additional data. This is called multiple regressions.
Suppose you measure a quantity *y* for several values of *x*1 and *x*2. Enter these into MATLAB at the command line, as follows:

x1 = [.2 .5 .6 .8 1.0 1.1]';
x2 = [.1 .3 .5 .9 1.1 1.4]';
y = [.17 .26 .28 .23 .27 .24]';

A model of this data is of the form

$$y = a0 + a1x1 + a2x2$$

Multiple regressions solves for unknown coefficients $a0$, $a1$, and $a2$ by minimizing the sum of the squares of the deviations of the data from the model (*least-squares fit*).

Construct and solve the set of simultaneous equations by forming the Vander monde matrix, X, and solving for the parameters by using the backslash operator:

$$X = [ones (size(x1)) x1 x2];$$

$$a = X\backslash y$$
$$a =$$
$$0.1018$$
$$0.4844$$
$$-0.2847$$

The least-squares fit model of the data is
$$Y = 0.1018 + 0.4844x1 - 0.2847x2$$

From Table II :-

```
>> x1=[28510 28300 26500 27165]';
>> x2=[5853200 5685000 5716510 5451320]';
>> y=[35345 34510 33120 33600]';
>> x=[ones(size(x1)) x1 x2];
>> a=x\y;
>> a
  a =
  1.0e+003 *
  2.5663
  0.0009
```

0.0000

```
>> format long
>> a
a =
  1.0e+003 *
  2.566279260095765
  0.000903995653067
  0.000001164490806.
```

TABLE-IV. COMPARISION AMONG ALL ADOPTED METHODS

USING NUMERICAL METHOD	USING MATLAB	USING EXCEL
I=2566.273559	I=1.0e+003* 2.566279260095765	I=2566.28154
X_s=0.903995465	X_s= 1.0e+003* 0.000903995653067	X_s=0.90399571
X_e=1.164492726*10^-3	X_e= 1.0e+003* 0.000001164490806	X_e=0.00116449

VI. ESTIMATION OF UNIT COSTS FOR STEAM AND ELECTRICAL DEMAND

Calculation for Unit cost of Steam & Electricity:-

From Financial Data of the company;

Cogeneration project Cost= Rs. 46.11 crore.

So,

A_{total} $= 46.11*10^7$ Rs.

C_{bs} $=$ Atotal / Sb

$=46.11*10^7/136575$

$= 3376.1669$ Rs/T

$C_{bs} * X_s = (3376.1669)*(0.903996)$

$=3052..04141$ Rs/T

Unit Cost of Steam = $C_{bs} * X_s$ = 3.052204 Rs/Kg.

Now,

$C_{bs} * X_e = (3376.1669)*(0.001164*10^3)$

$= 3929.858272$ Rs/Wh.

Unit Cost of Electricity = $C_{bs} * X_e$ =3.9298582 Rs/Kwh.

- **This cost may vary if bagasse is brought outside from nearby plant during off-season period as the plant is newly commissioned.**

Calculation of Relative % error in boiler steam:

Using values of I, Xs & Xe constants in the below equation,

$$S_b = I + X_sS_d + X_eE$$

we can find error monthly between calculated boiler steam production & actual boiler steam production.

$$\% \; error = \frac{(Sb\;measured) - (Sb\;calculated)}{(Sb\;measured)} * 100$$

Table-V. PERCENTAGE RELATIVE ERROR

S_b(measured)	S_b(calculated)	% Relative error
35345	35155.19309	0.537012052
34510	34769.48367	-0.751916459
33120	33178.71414	-0.177276992
33600	33471.33285	0.382937946

VII. CONCLUSION

Without a systematic method for evaluating the unit costs of steam and electricity delivered from a co-generation power plant, these figures can be very difficult to obtain. When a mathematical model is developed and fitted to data taken from the boiler logs, the unit costs of each utility can be accurately computed. The method of least squares allows the errors to be minimized between calculated and measured boiler steam delivery rates. The accuracy of this comparison

give assurance that the model is appropriate and that the unit costs have been arrived at correctly.

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