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Small Cosmological Constant from De Broglie-Bohm Quantum Theory

Jung-Jeng Huang

Abstract*—We propose a unified mechanism for generating a small cosmological constant through cascade transition in the history of the Universe in the context of de Broglie-Bohm quantum theory. In our previous work we studied the possible effects of trans-Planckian physics on the Bohm quantum trajectories of massless minimally coupled scalar field in de Sitter space. The result showed that for the Corley-Jacobson type dispersion relation with sextic correction, there exists a transition in the evolution of the quantum trajectory from well before horizon exit to well after horizon exit, providing a possible mechanism for generating a small cosmological constant. In this paper we obtain similar transitional behaviour for the Corley-Jacobson type dispersion relation with quartic correction. We find that if we compare the trans-Planckian effects on the Bohm quantum trajectories due to quartic and sextic corrections, the latter is much smaller than the former. We calculate explicitly the finite vacuum energy density due to fluctuations of the inflaton field and show how the cosmological constant reduces during the slow-roll inflation at the grand unification phase transition. Similar reduction mechanisms at the electroweak, quark-hadron and current accelerating phase transitions are also suggested to yield the current small value of the cosmological constant.

***Keywords*—cosmological constant, de Broglie-Bohm theoty, Schrödinger picture, trans-Planckian physics.**

I. INTRODUCTION

In the inflationary cosmology, the density perturbations which seed the present structures of our observed Universe [1] arise from the quantum fluctuations of scalar field about the standard Bunch-Davies (BD) vacuum state [2]. However, during inflation there is ambiguity in the notion of a vacuum state in quantum theory [3], and the choice of initial quantum vacuum state affects the predictions of inflation [4, 5].

For example, a deterministic hidden-variables theory such as the de Broglie-Bohm pilot-wave theory [6-10] allows the existence of vacuum states with non-standard or nonequilibrium field fluctuations [11, 12], which result in statistical predictions that deviate from those of quantum theory in the context of inflationary cosmology [13, 14]. Recently it has been shown that the quantum-to-classical transition of primordial cosmological perturbations can be explained easily and naturally in the context of the de Broglie-Bohm theory [15].

It is also well known that the inflationary cosmology suffers from a serious trans-Planckian problem [16, 17], which is whether the predictions of standard cosmology are insensitive to the effects of trans-Planckian physics.

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Jung-Jeng Huang, Department of Mechanical Engineering, Physics Division, Ming Chi University of Technology,

In fact, nonlinear dispersion relations such as the Corley-Jacobson (CJ) type were used to mimic the trans-Planckian effects on cosmological perturbations [16-18]. Recently these CJ type dispersion relations can be obtained naturally from Horava quantum gravity models [19-21]. Moreover, in several approaches to quantum gravity, the phenomenon of running spectral dimension of spacetime from the standard value of 4 in the infrared to a smaller value in the ultraviolet is associated with modified dispersion relations, which also include the CJ type dispersion relations [22, 23].

In our previous work [24-28] we used the lattice Schrödinger picture to study the free scalar field theory in de Sitter space, derived the wave functionals for the BD vacuum state and its excited states, and found the trans-Planckian effects on the Bohm quantum trajectories for the CJ type dispersion relation with sextic correction. The purpose of this paper is to study further the trans-Planckian effects on the Bohm quantum trajectories for the CJ type dispersion relation with quartic correction.

The paper is organized as follows. In Section 2, the pilot-wave theory of a generically coupled scalar field in de Sitter space is briefly reviewed in the lattice Schrödinger picture, and the de Broglie quantum trajectories for scalar field are given. In Section 3, we obtain the time evolution of vacuum state wave functional of massless minimally coupled scalar field during slow-roll inflation under the effects of trans-Planck physics. In Section 4, we apply the result of Section 3 to obtain the Bohm quantum trajectories through Bohm's dynamics. In Section 5, using the result of Section 4, we calculate the finite vacuum energy density and use the backreaction constraint to address the cosmological constant problem. Finally, the discussion and conclusion are presented in Section 6 and Section 7 respectively. Throughout this paper we will set $\hbar$ =c=1.

II. PILOT-WAVE SCALAR FIELD IN SCHRÖDINGER PICTURE

In this section, we begin by briefly reviewing how to define the pilot-wave theory of scalar field in de Sitter space in the lattice Schrödinger picture. The Lagrangian density for the generically coupled scalar field is

$$L = |g|^{\frac{1}{2}}\left\{\frac{1}{2}\left[g^{\mu\nu}(x)\phi(x)_{,\mu}\phi(x)_{,\nu}\right]-V(\phi)\right\}$$

$$V(\phi) = m^2\phi^2/2+\xi R\phi^2/2, \qquad (1)$$

where ϕ is a real scalar field, $V(\phi)$ is the potential , *m* is the mass of the scalar quanta, R is the Ricci scalar curvature, ξ is the coupling parameter, and g = det $g_{\mu\nu}$, μ,ν =0,1,…,d. For a spatially flat (1+d)-dimensional

Robertson-Walker spacetime with scale factor $a(t)$, we have

$$ds^2 = dt^2 - a^2(t)d^2x^i,\ i = 1,2,...,d,$$

$$L = a^d\left\{\frac{1}{2}\left[(\partial_0\phi)^2 - a^{-2}(\partial_i\phi)^2\right] - V(\phi)\right\}. \tag{2}$$

In the (1+d)-dimensional de Sitter space we have $a(t) = \exp(ht)$, where $h \equiv \dot{a}/a$ is the Hubble parameter which is a constant.

For d=1, in the lattice Schrödinger picture, we obtain from (2) the time-dependent functional Schrödinger equation in momentum space (for the details see [24-28])

$$H\psi = i\frac{\partial}{\partial t}\psi, \tag{3}$$

where $$H = 2\sum_{l=1}^{N/2}\sum_{r=1}^{2} H_{rl}, \tag{4}$$

$$H_{rl} = \frac{1}{2}p_{rl}^{\ 2} + \frac{1}{2}hp_{rl}\phi_{rl} + \frac{1}{2}a^{-2}\omega_l^{\ 2}\phi_{rl}^{\ 2} + \frac{1}{2}\left(m^2 + \xi R\right)\phi_{rl}^{\ 2}, \tag{5}$$

$$\psi[\phi_{rl}, t] = \prod_{l=1}^{N/2}\prod_{r=1}^{2}\psi_{rl}(\phi_{rl}, t) \equiv \prod_{rl}\psi_{rl}(\phi_{rl}, t). \tag{6}$$

Here $\omega_l \equiv (2/\varepsilon)\sin(l\pi/N)$, $\varepsilon = W/N$, W is the overall comoving spatial size of lattice, $\phi_l = \phi_{1l} + i\phi_{2l}$, $p_l = p_{1l} + ip_{2l}$, p_l is the conjugate momentum for ϕ_l, and the subscripts 1and 2 denote the real and imaginary parts respectively.

For each real mode ϕ_{rl}, we have

$$H_{rl}\psi_{rl} = i\frac{\partial}{\partial t}\psi_{rl},\ r=1,2, \tag{7}$$

$$-\frac{1}{2}\frac{\partial^2\psi_{rl}}{\partial\phi_{rl}^{\ 2}} + \frac{1}{2}\left[a^{-2}\omega_l^{\ 2} + \left(m^2 + \xi R\right) - \frac{1}{4}h^2\right]\phi_{rl}^{\ 2}\psi_{rl} = i\frac{\partial\psi_{rl}}{\partial t} \tag{8}$$

Note that (8) arises from the field quantization of the Hamiltonian (5) through the functional Schrödinger representation $\hat{\phi}_{rl} \to \phi_{rl}$, $\hat{p}_{rl} \to -i\partial/\partial\phi_{rl}$, where operators $\hat{\phi}_{rl}$ and $\hat{p}_{rl}$ satisfy the equal time commutation relations $\left[\hat{\phi}_{rl}, \hat{p}_{rl}\right] = i$. It governs the time evolution of the state wave functional $|\psi_{rl}\rangle$ of the Hamiltonian operator H_{rl} in the $\{|\phi_{rl}\rangle\}$ representation [28]. In terms of the conformal time τ defined by

$$d\tau = dt/a,\ \tau = -h^{-1}\exp(-ht) = -h^{-1}a^{-1},$$
$$-\infty < \tau < 0, \tag{9}$$

the normalized vacuum and its excited states are

$$\psi_{rl(n_{rl})}(\phi_{rl},\tau) = R_{(n_{rl})}(\phi_{rl},\tau)\exp\left(i\Theta_{(n_{rl})}(\phi_{rl},\tau)\right)$$
$$n_{rl} = 0,1,2,... \tag{10}$$

with the amplitude $R_{(n_{rl})}(\phi_{rl},\tau)$ and phase $\Theta_{(n_{rl})}(\phi_{rl},\tau)$

$$R_{(n_{rl})}(\phi_{rl},\tau) = \left[\frac{\sqrt{2h/\pi}}{\sqrt{\pi}2^n n_{rl}!\left|H_\nu^{(1)}\right|}\right]^{1/2} H_{(n_{rl})}(\eta_{rl})\exp(-\frac{1}{2}\eta_{rl}^{\ 2}) \tag{11}$$

$$\Theta_{(n_{rl})}(\phi_{rl},\tau) = -\frac{h\omega_l|\tau|}{2}\frac{\left(\left|H_\nu^{(1)}\right|\right)'}{\left|H_\nu^{(1)}\right|}\phi_{rl}^{\ 2} - (\frac{1}{2} + n_{rl})\int\frac{\frac{2}{\pi|\tau|}}{\left|H_\nu^{(1)}\right|^2}d\tau \tag{12}$$

Here η_{rl} is defined by $\eta_{rl} \equiv \left(\sqrt{2h/\pi}/\left|H_\nu^{(1)}\right|\right)\phi_{rl}$, $H_{(n_{rl})}(\eta_{rl})$ is the nth-order Hermite polynomial, $H_\nu^{(1)}(\omega_l|\tau|)$ is the Hankel function of the first kind of order ν, $\nu^2 = 1/4 - \left(m^2 + \xi R\right)/h^2$, and the prime in (12) denotes the derivative with respect to $\omega_l|\tau|$. The complete wave functionals can be written as $\psi_{[n]}[\phi_{rl},t] = \prod_{rl}\psi_{(n_{rl})}(\phi_{rl},t)$, where $[n] \equiv (n_i, n_j, \cdots)$ means that mode i is in the n_i excited state, mode j is in the n_j excited state, *etc*. For $n_{rl} = 0$, the ground state wave functional corresponds to the BD vacuum. For d=3, we have $\nu^2 = 9/4 - \left(m^2 + \xi R\right)/h^2$, $R = 12h^2$ and the mode index l in ω_l carries labels $(l_i, i = 1,2,3)$ which will be suppressed below.

For d=3, we get from equations (3)-(8) in the continuum limit ($\omega_l \to k$)

$$i\frac{\partial\psi}{\partial t} = \sum_{rk}\{-\frac{1}{2}\frac{\partial^2}{\partial\phi_{rk}^{\ 2}} + \frac{1}{2}\left[a^{-2}k^2 + \left(m^2 + \xi R\right) - \frac{9}{4}h^2\right]\phi_{rk}^{\ 2}\}\psi, \tag{13}$$

which implies the continuity equation

$$\frac{\partial|\psi|^2}{\partial t} + \sum_{rk}\{\frac{\partial}{\partial\phi_{rk}}\left[|\psi|^2\frac{\partial\Theta}{\partial\phi_{rk}}\right]\} = 0 \tag{14}$$

and the de Broglie velocity field

$$\frac{d\phi_{rk}}{dt} = \frac{\partial\Theta}{\partial\phi_{rk}}, \tag{15}$$

where $\psi = |\psi|\exp[i\Theta]$. For a single mode ϕ_{rk}, we have $\psi_{rk} = |\psi_{rk}|\exp[i\Theta_{rk}]$ with $\Theta = \sum_{rk}\Theta_{rk}$, the continuity equation

$$\frac{\partial|\psi_{rk}|^2}{\partial t} + \frac{\partial}{\partial\phi_{rk}}\left(|\psi_{rk}|^2\frac{\partial\Theta_{rk}}{\partial\phi_{rk}}\right) = 0, \tag{16}$$

and the de Broglie velocity field

$$\frac{d\phi_{rk}}{dt} = \frac{\partial\Theta_{rk}}{\partial\phi_{rk}}. \tag{17}$$

Here ψ is interpreted as a physical field in field configuration space, guiding the evolution of ϕ_{rk} through (13) and (17). Substituting (12) into (17) and using τ gives

$$\frac{d\phi_{rk}}{d\tau} = -k\frac{\left(\left|H_\nu^{(1)}(k|\tau|)\right|\right)'}{\left|H_\nu^{(1)}(k|\tau|)\right|}\phi_{rk}\,, \tag{18}$$

which yields the quantum trajectory

$$\phi_{rk}(z) = C\left|H_\nu^{(1)}(z)\right|, \tag{19}$$

where $z \equiv k|\tau| = (k/a)/h$ is the ratio of physical wave number $k_{phys} \equiv k/a$ to the inverse of Hubble radius, and C is an integration constant.

III. Effects of Trans-Planckian Physics on Vacuum Wave Functional

To study further the effects of trans-Planckian physics on the evolution of vacuum state, we use the CJ type dispersion relations

$$\omega^2(k/a) = k^2\left[1 + b_s\left(\frac{k}{aM}\right)^{2s}\right], \tag{20}$$

where M is a cutoff scale, s is an integer, and b_s is an arbitrary coefficient [16-18]. In what follows, we focus on the CJ type dispersion relation (20) with $s=1$ and $b_1 > 0$, and consider the massless minimally coupled ($\nu = 3/2$) scalar field in the slow-roll inflation. Then using $z = k|\tau| = k/ah$, (13) becomes

$$i\frac{\partial\psi}{\partial t} = \sum_{rk}\left\{-\frac{1}{2}\frac{\partial^2}{\partial\phi_{rk}^2} + \frac{1}{2}\left[z^2\left(1+\sigma^2 z^2\right)h^2 - \frac{9}{4}h^2\right]\phi_{rk}^2\right\}\psi\,, \tag{21}$$

where $\sigma^2 \equiv b_1(h/M)^2$, and the ground state wave functional of (21) becomes

$$\psi_{(0)} = \prod_{rk} A_{k(0)}(\tau)\exp\left(-\frac{1}{2}B_k(\tau)a^{-1}\phi_{rk}^2\right), \tag{22}$$

where $A_{k(0)}(\tau)$ and $B_k(\tau)$ satisfy

$$A_{k(0)}(\tau) = \exp\left[-i\frac{1}{2}\int B_k(\tau)d\tau + const\right], \tag{23}$$

$$B_k^2(\tau) - i\left[\frac{dB_k(\tau)}{d\tau} + \frac{B_k(\tau)}{\tau}\right] - \left[k^2(1+\sigma^2 z^2) - \frac{9}{4\tau^2}\right] = 0\,. \tag{24}$$

In region I where $k_{phys} \equiv k/a > M$, i.e. $z > M/h$, the dispersion relation can be approximated by $\omega^2(k/a) \approx k^2\sigma^2 z^2$, and the corresponding wave functional for the initial BD vacuum state is [28]

$$\psi_{(0)}^{\mathrm{I}} = \prod_{rk} A_{k(0)}^{\mathrm{I}}(\tau)\exp\left(-\frac{1}{2}B_k^{\mathrm{I}}(\tau)a^{-1}\phi_{rk}^{\mathrm{I}2}\right),$$

$$A_{k(0)}^{\mathrm{I}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k^{\mathrm{I}}(\tau)d\tau + const\right], \tag{25}$$

$$B_k^{\mathrm{I}}(\tau) = \frac{\frac{4}{\pi|\tau|}}{\left|H_{3/4}^{(1)}\right|^2} - i\frac{\omega_l}{2}\frac{\left(\left|H_{\bar\nu}^{(1)}\right|^2\right)'}{\left|H_{3/4}^{(1)}\right|^2}\sigma z\,, \tag{26}$$

where the prime in (26) denotes the derivative with respect to $\sigma z^2/2$.

On the other hand, in region II where $k_{phys} \equiv k/a < M$, i.e. $z < M/h$, linear relation recovers $\omega^2 \cong k^2$, and the corresponding wave functional for the non-BD vacuum state is [28]

$$\psi_{(0)}^{\mathrm{II}} = \prod_{rk} A_{k(0)}^{\mathrm{II}}(\tau)\exp\left(-\frac{1}{2}B_k^{\mathrm{II}}(\tau)a^{-1}\phi_{rk}^{\mathrm{II}2}\right),$$

$$A_{k(0)}^{\mathrm{II}}(\tau) = \exp\left[-i\frac{1}{2}\int B_k^{\mathrm{II}}(\tau)d\tau + const\right], \tag{27}$$

$$B_k^{\mathrm{II}}(\tau) = \frac{\frac{2}{\pi|\tau|}}{\left(\left|C_1^{\mathrm{II}}\right|^2 + \left|C_2^{\mathrm{II}}\right|^2\right)\left|H_{3/2}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1^{\mathrm{II}}C_2^{\mathrm{II}*}\left(H_{3/2}^{(1)}\right)^2\right]}$$

$$-i\frac{k}{2}\frac{\left\{\left(\left|C_1^{\mathrm{II}}\right|^2 + \left|C_2^{\mathrm{II}}\right|^2\right)\left|H_{3/2}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1^{\mathrm{II}}C_2^{\mathrm{II}*}\left(H_{3/2}^{(1)}\right)^2\right]\right\}'}{\left(\left|C_1^{\mathrm{II}}\right|^2 + \left|C_2^{\mathrm{II}}\right|^2\right)\left|H_{3/2}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1^{\mathrm{II}}C_2^{\mathrm{II}*}\left(H_{3/2}^{(1)}\right)^2\right]}\,, \tag{28}$$

where the prime in (28) denotes the derivative with respect to z, and the constants C_1^{II} and C_2^{II} satisfy $\left|C_1^{\mathrm{II}}\right|^2 - \left|C_2^{\mathrm{II}}\right|^2 = 1$. Let τ_c be the time when the modified dispersion relations take the standard linear form. Then $\sigma^2 z_c^2 = 1$ where $z_c = k|\tau_c| = M/b_1^{1/2}h$ >>1 for $b_1 \sim 1$. The constants C_1^{II} and C_2^{II} can be obtained by the following matching conditions at τ_c for the two wave functionals (25) and (27)

$$\psi_{(0)}^{\mathrm{I}}\Big|_{Z_C} = \psi_{(0)}^{\mathrm{II}}\Big|_{Z_C}\,, \tag{29}$$

$$\frac{d\psi_{(0)}^{\mathrm{I}}}{dz}\Big|_{Z_C} = \frac{d\psi_{(0)}^{\mathrm{II}}}{dz}\Big|_{Z_C}\,, \tag{30}$$

which can also be rewritten respectively as

$$\mathrm{Re}\left(B_k^{\mathrm{I}}\right)\Big|_{Z_C} = \mathrm{Re}\left(B_k^{\mathrm{II}}\right)\Big|_{Z_C}\,, \tag{31}$$

$$\frac{d\,\mathrm{Re}\left(B_k^{\mathrm{I}}\right)}{dz}\Big|_{Z_C} = \frac{d\,\mathrm{Re}\left(B_k^{\mathrm{II}}\right)}{dz}\Big|_{Z_C}\,, \tag{32}$$

by requiring $B_k^{\mathrm{I}} = B_k^{\mathrm{II}}$, $\phi_{rk}^{\mathrm{I}} = \phi_{rk}^{\mathrm{II}}$, $A_{k(0)}^{\mathrm{I}} = A_{k(0)}^{\mathrm{II}}$ when $z = z_c$.

Using $\left|H_{3/4}^{(1)}(\sigma z^2/2)\right|^2 = (4/\pi\sigma z^2)(1 + 5/8\sigma^2 z^4 + \ldots)$

$\approx 4/\pi\sigma z^2$ with $\sigma = z_c^{-1}$, $z_c >> 1$ and $\left|H_{3/2}^{(1)}(z)\right|^2 = z^{-3}(1+z^2)$, we have from (26), (28), and (31)

$$1 = \left|C_1^{\mathrm{II}}\right|^2 + \left|C_2^{\mathrm{II}}\right|^2 + 2\left|C_1^{\mathrm{II}}\right|\left|C_2^{\mathrm{II}}\right|\cos(2z_c - \theta), \tag{33}$$

where we choose $C_1^{\mathrm{II}} = \left|C_1^{\mathrm{II}}\right|$ and $C_2^{\mathrm{II}} = \left|C_2^{\mathrm{II}}\right|\exp(i\theta)$, and θ is a relative phase parameter. Then from (33) and $\left|C_1^{\mathrm{II}}\right|^2 - \left|C_2^{\mathrm{II}}\right|^2 = 1$ we have

$$\left|C_1^{\mathrm{II}}\right| = \csc(2z_c - \theta), \; \left|C_2^{\mathrm{II}}\right| = -\cot(2z_c - \theta), \tag{34}$$

where $\sin(2z_c - \theta) > 0$, $\cos(2z_c - \theta) < 0$. Substituting (26) and (28) into (32) and keeping terms up to order $1/z_c$ on the right-hand side of (32), we obtain

$$\frac{1}{z_c} = \left|C_1^{\mathrm{II}}\right|\left|C_2^{\mathrm{II}}\right|\cos(2z_c - \theta)\frac{8}{z_c} + 4\left|C_1^{\mathrm{II}}\right|\left|C_2^{\mathrm{II}}\right|\sin(2z_c - \theta). \tag{35}$$

Using (34) in (35) gives

$$\cot(2z_c - \theta) = -\frac{1}{4z_c} \quad \text{or} \quad \cot(2z_c - \theta) = -\frac{z_c}{2} + \frac{1}{4z_c}. \tag{36}$$

Here we choose $\cot(2z_c - \theta) = -1/4z_c$, so that $\left|C_2^{\mathrm{II}}\right|$ is small for $z_c >> 1$ to avoid an unacceptably large backreaction on the background geometry. Then we have

$$\left|C_2^{\mathrm{II}}\right| \cong \frac{1}{4z_c}, \; \left|C_1^{\mathrm{II}}\right| = \sqrt{1 + \left|C_2^{\mathrm{II}}\right|^2} \cong 1 + \frac{1}{32z_c^2} \cong 1, \tag{37}$$

or

$$\sin(2z_c - \theta) \cong 1, \; \cos(2z_c - \theta) \cong -\frac{1}{4z_c}. \tag{38}$$

IV. BOHM QUANTUM TRAJECTORY

In Section 2, we defined the pilot-wave scalar field theory through de Broglie's first-order dynamics (13) and (17). Using the result about the evolution of vacuum state in Sec. 3, we can further define it through Bohm's second-order dynamics (21) and (39):

$$\frac{d^2\phi_{rk}}{dt^2} = -\frac{\partial}{\partial\phi_{rk}}(V + Q). \tag{39}$$

Here the classical potential V and the so-called 'quantum potential' Q are given by

$$V = \sum_{rk}\frac{1}{2}\left[z^2\left(1 + \sigma^2 z^2\right)h^2 - 9h^2/4\right]\phi_{rk}^{\;2}, \tag{40}$$

$$Q = -\sum_{rk}\frac{1}{2\left|\psi_{(0)}\right|}\frac{\partial^2\left|\psi_{(0)}\right|}{\partial\phi_{rk}^{\;2}}, \tag{41}$$

where $\psi_{(0)}$ is given by (22)-(24) and $\left|\psi_{(0)}\right|$ is given by the continuum limit of (11) for $n_{rl} = 0$. Note that Bohm's dynamics in general yields more possible quantum trajectories than de Broglie's dynamics does [27], and this distinction between Bohm's and de Broglie's dynamics was also emphasized recently by Valentini [29]. This is what we expect, because Bohm regarded (39) as the law of motion, with the de Broglie guidance equation (17) added as a constraint on the initial momenta.

In region I, the classical potential V in (40) becomes

$$V = \sum_{rk}\frac{1}{2}\left(\sigma^2 z^4 h^2 - 9h^2/4\right)\phi_{rk}^{\;\mathrm{I}2}, \tag{42}$$

and from (41), (25) and (26) the quantum potential Q becomes

$$Q = \sum_{rk}\left(-2\frac{(2h/\pi)^2}{\left|H_{3/4}^{(1)}\right|^4}\phi_{rk}^{\;\mathrm{I}2} + \frac{2h/\pi}{\left|H_{3/4}^{(1)}\right|^2}\right). \tag{43}$$

Substituting (42) and (43) in (39) and using $d\tau = dt/a$ and $z = k|\tau| = k/ah$ gives

$$z^2\frac{d^2\phi_{rk}^{\;\mathrm{I}}}{dz^2} + z\frac{d\phi_{rk}^{\;\mathrm{I}}}{dz} + \left[\sigma^2 z^4 - \frac{9}{4} - \frac{16}{\pi^2}\left|H_{3/4}^{(1)}\right|^{-4}\right]\phi_{rk}^{\;\mathrm{I}} = 0. \tag{44}$$

On the other hand, in region II, the potentials V and Q in (39) become respectively

$$V = \sum_{rk}\frac{1}{2}\left(z^2 h^2 - 9h^2/4\right)\phi_{rk}^{\;\mathrm{II}2}, \tag{45}$$

$$Q = \sum_{rk}\left(-\frac{1}{2}\frac{(2h/\pi)^2}{\left|H_{3/2}^{(1)}\right|_{md}^4}\phi_{rk}^{\;\mathrm{II}2} + \frac{1}{2}\frac{2h/\pi}{\left|H_{3/2}^{(1)}\right|_{md}^2}\right), \tag{46}$$

where $\left|H_{3/2}^{(1)}\right|_{md}$ means $\left|H_{3/2}^{(1)}\right|$ modified according to

$$\left|H_{3/2}^{(1)}\right|_{md} \equiv \left\{\left(\left|C_1\right|^2 + \left|C_2\right|^2\right)\left|H_{3/2}^{(1)}\right|^2 + 2\,\mathrm{Re}\left[C_1 C_2^*\left(H_{3/2}^{(1)}\right)^2\right]\right\}^{1/2}.$$

Then the quantum trajectory $\phi_{rk}^{\;\mathrm{II}}$ satisfies

$$z^2\frac{d^2\phi_{rk}^{\;\mathrm{II}}}{dz^2} + z\frac{d\phi_{rk}^{\;\mathrm{II}}}{dz} + \left[z^2 - \frac{9}{4} - \frac{4}{\pi^2}\left|H_{3/2}^{(1)}\right|_{md}^{-4}\right]\phi_{rk}^{\;\mathrm{II}} = 0. \tag{47}$$

Using $\left|H_{3/4}^{(1)}\left(\sigma z^2/2\right)\right|^2 = \left(4/\pi\sigma z^2\right)\left(1 + 5/8\sigma^2 z^4 + \ldots\right)$ in region I, (44) becomes approximately

$$z^2\frac{d^2\phi_{rk}^{\;\mathrm{I}}}{dz^2} + z\frac{d\phi_{rk}^{\;\mathrm{I}}}{dz} - \phi_{rk}^{\;\mathrm{I}} = 0. \tag{48}$$

The general solution of (48) is

$$\phi_{rk}^{\;\mathrm{I}}(z) = \bar{C}_1^{\mathrm{I}} z^{-1} + \bar{C}_2^{\mathrm{I}} z, \tag{49}$$

where $\bar{C}_1^{\mathrm{I}}$ and $\bar{C}_2^{\mathrm{I}}$ are constants to be fixed by choosing suitable initial conditions at an arbitrary initial time τ_0 for $\phi_{rk}^{\;\mathrm{I}}$. Here we choose $\bar{C}_1^{\mathrm{I}} \neq 0$ and $\bar{C}_2^{\mathrm{I}} = 0$ so that the first term $\bar{C}_1^{\mathrm{I}} z^{-1}$ in (49) corresponds to (19) with

$\left|H_{3/4}^{(1)}\left(\sigma z^2/2\right)\right| \approx (2/\sqrt{\pi\sigma})z^{-1}$. On the other hand, in region II, $\left|H_{3/2}^{(1)}\right|_{md}$ becomes

$$\left|H_{3/2}^{(1)}\right|_{md} = \left|H_{3/2}^{(1)}\right| \Big\{ \left|C_1^{\text{II}}\right|^2 + \left|C_2^{\text{II}}\right|^2 + 2\left|C_1^{\text{II}}\right|\left|C_2^{\text{II}}\right| \left[\cos(2z-\theta)\frac{z^2-1}{z^2+1} - \sin(2z-\theta)\frac{2z}{1+z^2} \right] \Big\} \tag{50}$$

which reduces to $\left|H_{3/2}^{(1)}\right|$ for $z \to z_c >> 1$ (well before horizon exit) by using (33). Hence, ϕ_{rk}^{II} satisfies

$$z^2 \frac{d^2\phi_{rk}^{\text{II}}}{dz^2} + z\frac{d\phi_{rk}^{\text{II}}}{dz} + \left[z^2 - \frac{9}{4} - \frac{4}{\pi^2}\left|H_{3/2}^{(1)}\right|^{-4} \right]\phi_{rk}^{\text{II}} = 0. \tag{51}$$

The general asymptotic series solution of (51) is [27]

$$\phi_{rk}^{\text{II}}(z) = \overline{C}_1^{\text{II}} z^{-1/2}\left(1 + \frac{1}{2}z^{-2} - \frac{1}{8}z^{-4} + \cdots\right) + \overline{C}_2^{\text{II}} z^{1/2}\left(1 + \frac{3}{2}z^{-2} + \frac{1}{24}z^{-4} + \cdots\right). \tag{52}$$

For $z \to z_c >> 1$, (52) reduces to

$$\phi_{rk}^{\text{II}}(z) \approx \overline{C}_1^{\text{II}} z^{-1/2} + \overline{C}_2^{\text{II}} z^{1/2}. \tag{53}$$

Substituting (49) and (53) into the matching conditions at z_c for ϕ_{rk}^{I} and ϕ_{rk}^{II}

$$\phi_{rk}^{\text{I}}\Big|_{z_C} = \phi_{rk}^{\text{II}}\Big|_{z_C}, \quad \frac{d\phi_{rk}^{\text{I}}}{dz}\Big|_{z_C} = \frac{d\phi_{rk}^{\text{II}}}{dz}\Big|_{z_C}, \tag{54}$$

we obtain

$$\overline{C}_1^{\text{II}} = \frac{3}{2}\overline{C}_1^{\text{I}} z_c^{-1/2}, \quad \overline{C}_2^{\text{II}} = -\frac{1}{2}\overline{C}_1^{\text{I}} z_c^{-3/2}. \tag{55}$$

From (53) and (55) we see that as z decreases from z_c to 1, $\overline{C}_1^{\text{II}} z^{-1/2}$ becomes the dominant term, *i.e.*

$$\phi_{rk}^{\text{II}}(z) \approx \overline{C}_1^{\text{II}} z^{-1/2}. \tag{56}$$

Furthermore, for $z << 1$ (well after horizon exit), (50) reduces to $\left|H_{3/2}^{(1)}\right|$ by using (37) and $z_c >> 1$, and the general power series solution of (51) is [27]

$$\phi_{rk}^{\text{II}}(z) = \hat{C}_1^{\text{II}} z^{-3/2}\left(1 + \frac{1}{2}z^{2} - \frac{1}{8}z^{4} + \cdots\right) + \hat{C}_2^{\text{II}} z^{3/2}\left(1 - \frac{1}{10}z^{2} + \frac{1}{280}z^{4} + \cdots\right). \tag{57}$$

For $z << 1$, (57) reduces to

$$\phi_{rk}^{\text{II}}(z) \approx \hat{C}_1^{\text{II}} z^{-3/2}, \tag{58}$$

which corresponds to (19) with $z << 1$. Here we have $\hat{C}_1^{\text{II}} = \overline{C}_1^{\text{II}}$. (See the argument in the discussion in Section 5 of [28].) Requiring ϕ_{rk}^{II} to be continuous at $z = 1$, we have from (55), (56) and (58)

$$\hat{C}_1^{\text{II}} = \overline{C}_1^{\text{II}} = \frac{3}{2}\overline{C}_1^{\text{I}} z_c^{-1/2}. \tag{59}$$

Since for d=3 ϕ_{rk} contains a factor $a^{3/2}$ which is proportional to $z^{-3/2}$, we use a field redefinition $u_{rk} \equiv a^{-3/2}\phi_{rk}$ and $a = (k/h)z^{-1}$ to rewrite (49) and (58) as

$$u_{rk}^{\text{I}} = \left(\frac{k}{h}\right)^{-3/2} \overline{C}_1^{\text{I}} z^{1/2}, u_{rk}^{\text{II}} = \frac{3}{2}\left(\frac{k}{h}\right)^{-3/2} \overline{C}_1^{\text{I}} z_c^{-1/2}. \tag{60}$$

Thus we see from (60) that for fixed k and $z_c >> 1$, as z decreases from $z = z_c >> 1$ to $z << 1$, the scalar field decreases from one large value to a much smaller constant which is a factor $3/2z_c$ less than the field value at $z = z_c$, *i.e.*, there exists a transition in the time evolution of the quantum trajectory of scalar field. We also note that if we consider only the CJ type dispersion relation with sextic correction, then (60) is replaced by [28]

$$u_{rk}^{\text{I}} = \left(\frac{k}{h}\right)^{-3/2} \overline{C}_1^{\text{I}}, \ u_{rk}^{\text{II}} = \frac{2}{\bar{z}_c}\left(\frac{k}{h}\right)^{-3/2} \overline{C}_1^{\text{I}}, \tag{61}$$

where $\bar{z}_c = k|\bar{\tau}_c| = M/b_2^{1/4}h$. Hence as z decreases from $z = \bar{z}_c >> 1$ to $z << 1$, the scalar field decreases from one large constant to a much smaller constant which is a factor $2/\bar{z}_c$ less than the field value at $z = \bar{z}_c$. Comparing (60) with (61) for scalar field values, we find that for $b_1 \approx b_2$, $z_c \approx \bar{z}_c >> 1$, as z decreases from $z = z_c \approx \bar{z}_c >> 1$ to $z << 1$, the former is almost larger than the latter by a factor $z_c^{1/2} \approx \bar{z}_c^{1/2}$ during the evolution of the quantum trajectories. Thus if we compare the trans-Planckian effects of both quartic and sextic corrections on the quantum trajectories, the latter is much smaller than the former.

V. BACKREACTION AND SMALL COSMOLOGICAL CONSTANT

Note that the vacuum energy density due to the fluctuations of the inflaton field with $k < k_{\max}$ is given by [1]

$$\rho_{vac} = \frac{1}{2\pi^2}\int_0^{k_{\max}} \left(\frac{k}{a}\right)^4 \frac{dk}{k} = \frac{1}{8\pi^2} k_{phys(\max)}^{\ \ 4} = \frac{1}{8\pi^2} M^4, \tag{62}$$

where M is the momentum cutoff. Because having a non standard dispersion relation is equivalent to considering non-vacuum quantum states for the perturbations [30], the finite energy density due to the inflaton particles after the subtraction of zero-point energy is given by [31, 32]

$$\rho_{vac} = \frac{1}{2\pi^2}\int_0^{k_{\max}} < n_k > \left(\frac{k}{a}\right)^4 \frac{dk}{k}, \tag{63}$$

where $< n_k >$ is the occupation number of the modes with the momentum k, which is equal to $\left|C_2^{\text{II}}\right|^2$ in our formulation (see Section 3). Then, for the case of $s=1$ and $b_1>0$, using $\left|C_2^{\text{II}}\right| \cong 1/4z_c$ in (37), we have $< n_k > \cong 1/16{z_c}^2$. From (62), (63) and $z_c = M/{b_1}^{1/2}h$, we obtain

$$\rho_{vac(s=1)} = \frac{M^4}{128\pi^2{z_c}^2} = \frac{b_1}{128\pi^2}M^2h^2. \tag{64}$$

Thus, we see that there is no backreaction problem if the energy density due to fluctuations of the inflaton field is smaller than that due to the inflaton potential, *i.e.*

$$\rho_{vac(s=1)} = \frac{b_1}{128\pi^2}M^2h^2 < V(\phi). \tag{65}$$

Within the slow-roll approximation, substituting $V(\phi) \cong 3{M_{Pl}}^2h^2/8\pi$ ($M_{Pl} =$ 1.22×10^{19} GeV is the Planck mass) in (65) leads to $b_1 < 48\pi({M_{Pl}}^2/M^2)$. For $M \sim M_{Pl}$, the constraint on the parameter b_1 is $b_1 < 1.5\times10^2$.

On the other hand, for the case of $s=2$ and $b_2>0$, we have $\left|C_2^{\text{II}}\right| \cong 1/2\bar{z}_c$, $< n_k > \cong 1/4{\bar{z}_c}^2$, and

$$\rho_{vac(s=2)} = \frac{M^4}{32\pi^2{\bar{z}_c}^4} = \frac{b_2}{32\pi^2}h^4, \tag{66}$$

where $\bar{z}_c = M/{b_2}^{1/4}h$ and the additional factor ${\bar{z}_c}^2$ in the denominator comes from comparing (60) with (61) for scalar field values. There is no backreaction problem if

$$\rho_{vac(s=2)} = \frac{b_2}{32\pi^2}h^4 < V(\phi). \tag{67}$$

Using $V(\phi) \cong 3{M_{Pl}}^2h^2/8\pi$ in (67) leads to $b_2 < 12\pi({M_{Pl}}^2/h^2)$. For $M_{Pl} = 1.22\times10^{19}$ GeV and $h \sim 10^{15}$ GeV (the Hubble constant during inflation which is close to the grand unification scale), the constraint on the parameter b_2 is about $b_2 < 5.6\times10^9$. Therefore, if we consider standard linear dispersion relation with both quartic and sextic corrections, then the energy density due to fluctuations of the inflaton field can be rewritten as

$$\rho_{vac}(h) = \rho_{vac}(h=0) + \rho_{vac(s=0)}(h) + \rho_{vac(s=1)}(h) + \rho_{vac(s=2)}(h)$$
$$= c_0 + c_2M^2h^2 + c_4h^4, \tag{68}$$

where $\rho_{vac}(h=0) \equiv c_0$ is the flat space contribution (62) before the beginning of inflation, $\rho_{vac(s=0)} = \dfrac{-29}{960\pi^2}h^4$ is the BD vacuum energy density of the massless minimally coupled inflaton field with standard linear dispersion relation [3], and $c_2 = b_1/128\pi^2$, $c_4 = (30b_2 - 29)/960\pi^2$. For $M \sim M_{Pl} >> h$, $b_1 \sim 1$, and $b_2 \sim 1$, we have $c_2 \sim 10^{-3}$ and (68) becomes

$$\rho_{vac}(h) \approx c_0 + c_2M^2h^2 . \tag{69}$$

Since a constant scalar field corresponds to a cosmological constant Λ, the transition of scalar field could be interpreted as a transition of the Universe from a large Λ to a small Λ, thus providing a possible mechanism for generating a small Λ in the Bohmian approach to inflationary cosmology. From $\rho_{vac}(h=0) = M^4/8\pi^2$, (60) and (61), we expect that while z decreases from $z = z_c >> 1$ to $z << 1$, the cosmological constant $\Lambda = 8\pi\rho_{vac}(h)/{M_{Pl}}^2$ decreases as

$$(1/\pi)(M^4/{M_{Pl}}^2) \rightarrow 8\pi c_2\left(M^2/{M_{Pl}}^2\right)h^2 \rightarrow (c_4/\pi)(h^4/{M_{Pl}}^2). \tag{70}$$

Such a reduction happens during the early inflationary era when the Hubble parameter is close to the grand unification scale $h \sim 10^{15}$ GeV.

Because current astronomical observations [33] indicate that the present value of vacuum energy density and cosmological constant are ${\rho_{vac}}^0 \sim 2.5\times10^{-47}$ GeV^4 and $\Lambda^0 \sim 4.2\times10^{-84}$ GeV^2, we propose that such a small current value of Λ in principle could be generated through a series of similar reductions from cascade transition (electroweak phase transition, quark-hadron phase transition, *etc*.) in the history of the Universe. That is, barring the inflation era associated with the grand unification scale, there are three additional little (or tepid) inflation eras in a series of phase transitions (see [34, 35] for the little inflation at the cosmological QCD phase transition). Thus, during the inflation at the electroweak phase transition, the Hubble parameter is close to the electroweak scale $h' \sim 10^2$ GeV, and Λ reduces further as $(1/\pi)(M'^4/{M_{Pl}}^2) \rightarrow$ $8\pi c_2\left(M'^2/{M_{Pl}}^2\right)h'^2 \rightarrow (c_4/\pi)(h'^4/{M_{Pl}}^2)$, where $M'^4 = c_4h^4$; during the inflation at the quark-hadron phase transition , the Hubble parameter is close to the QCD scale $h'' \sim 200$ MeV, and Λ reduces further as $(1/\pi)(M''^4/{M_{Pl}}^2) \rightarrow 8\pi c_2\left(M''^2/{M_{Pl}}^2\right)h''^2 \rightarrow (c_4/\pi)(h''^4/{M_{Pl}}^2)$, where $M''^4 = c_4h'^4$; during the inflation at the current accelerating phase transition, the Hubble parameter is close to the dark energy scale $h''' \sim 2.2\times10^{-3}$ eV, Λ reduces further as $(1/\pi)(M'''^4/{M_{Pl}}^2) \rightarrow 8\pi c_2\left(M'''^2/{M_{Pl}}^2\right)h'''^2 \rightarrow (c_4/\pi)(h'''^4/{M_{Pl}}^2)$, where $M'''^4 = c_4h''^4$. Requiring $(c_4/\pi)(h'''^4/{M_{Pl}}^2) \sim 4.2\times10^{-84}$ GeV^2 leads to $c_4 \sim 84$ or $b_2 \sim 2.7\times10^4$

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VI. DISCUSSION

Since we view the cosmological constant from the point of view of vacuum energy density in the context of the de Broglie-Bohm pilot-wave theory, we would like to elaborate on some important points of our proposal in this section.

(i) Note that the quantum potential Q plays an important role in the evolution of the quantum trajectory of the inflaton field. From (39) and (51), we see that for $z = z_c >> 1$ and the mode $\phi_{rk}^{\ II}$, the quantum force $\left(4h^2/\pi^2\right)\left|H_{3/2}^{\ (1)}\right|^{-4}\phi_{rk}^{\ II}$ arising from the quantum potential Q through $-\partial Q/\partial\phi_{rk}^{\ II}$ approximately cancels theclassical force $-\left(z^2h^2 - 9h^2/4\right)\phi_{rk}^{\ II}$ arising from the classical potential V through $-\partial V/\partial\phi_{rk}^{\ II}$, while for $z << 1$ the quantum force becomes negligible with respect to the classical force. Therefore, as z decreases from z_c to 0, there is a quantum-to-classical transition. In this context, we can neglect the quantum zero-point contributions in the calculation of the vacuum energy density such as (68).

(ii) The vacuum energy density also receives contributions from condensates associated with spontaneous symmetry breaking in the Standard Model. The parameter c_4 in (70) is positive if $b_2 > 29/30 \sim 0.97$, which is a condition quite easy to hold from the analysis of backreaction constraint on the parameter b_2 in Section 5. Thus, for $c_4 > 0$, the electroweak condensate gives positive $\rho_{vac} \sim c_4\ h'^4 \sim c_4 \times 10^8$ GeV ^{4} in our proposed picture, whereas the electroweak condensate gives negative $\rho_{vac} \sim\ -1.2\times 10^8$ GeV ^{4} in the Standard Model [36]. Similarly, the QCD condensate gives positive $\rho_{vac} \sim c_4\ \ h'^4 \sim c_4 \times (200)^4$ MeV ^{4} in our proposed picture, whereas the QCD condensate gives negative $\rho_{vac} \sim -(200)^4$ MeV ^{4} in the Standard Model.

(iii) In our proposed picture, we merely assume the existence of a symmetry breaking scale 2.2×10^{-3} eV in the last little inflation. The vacuum energy density associated with dark energy is characterized by a scale around 2.2×10^{-3} eV, which is close to the range of possible light neutrino masses. Some possible explanations for this relation include "growing neutrinos" scenario [37] and spin model neutrinos [38].

(iv) The vacuum energy density (68) in an inflationary universe can be extended to a generic dynamical model [36]

$$\rho_{vac}(H) = \bar{c}_0 + \bar{c}_2 H^2 + \bar{c}_4 H^4, \qquad (71)$$

where $\rho_{vac}(H)$ is the running vacuum energy density in an expanding universe, the Hubble rate H is the natural running scale in the cosmological scale, and $\rho_{vac}(H)$ is expected to satisfy a general renormalization group equation of the form

$$\frac{d\rho_{vac}(H)}{d\ln H^2} = \frac{1}{(4\pi)^2}\sum_i\left[a_i M_i^{\ 2} H^2 + b_i H^4\right] \qquad (72)$$

(M_i are the masses of participating particles). Here the coefficients $\bar{c}_2$ and $\bar{c}_4$ in (71) are related the coefficients a_i and b_i in (72) as follows:

$$\bar{c}_2 = \frac{1}{16\pi^2}\sum_{i=b,f} a_i M_i^{\ 2}\ ,\ \ \bar{c}_4 = \frac{1}{32\pi^2}\sum_{i=b,f} b_i\ , \qquad (73)$$

where independent contributions from bosons and fermions with different multiplicities are assumed for the more general case. For the special case of a single inflaton scalar field (*i*=inflaton), from (68) and (73) we have $a_i = (b_1/8)(M^2/M_i^{\ 2})$, $b_i = b_2 - 29/30$. In the slow-roll inflation, the mass of inflaton M_i is required to satisfy $M_i << h$.

(v) Note that (62) is calculated in the comoving de Sitter coordinates. If we make the transformation to the static de Sitter coordinates, then for an observer situated at the origin, there is a cosmological horizon at Hubble radius $1/h$ which marks the boundary of observable universe. Thus, it is expected that ρ_{vac} in (62) becomes $M^4/8\pi^2 - h^4/8\pi^2$, where the term $h^4/8\pi^2$ arises from the lower limit of integration in (62). Thus, for $M \sim M_{Pl} >> h$, the results in Section 5 are unchanged essentially if we change reference frame.

VII. CONCLUSION

In the lattice Schrödinger picture, we have considered the de roglie-Bohm pilot-wave theory of a generically coupled free real scalar field in de Sitter space. To investigate the possible effects of trans-Planckian physics on the quantum trajectories of the vacuum state of scalar field, we focused on the massless minimally coupled scalar field, and considered the CJ type dispersion relation with quartic correction.

Our previous work showed that for the CJ type dispersion relation with sextic correction, there exists a transition in the evolution of the quantum trajectory from well before horizon exit to well after horizon exit, providing a possible mechanism for generating a small cosmological constant. In this paper, we have found that for the CJ type dispersion relation with quartic correction , there exists a similar transition. We note that if we analyze the trans-Planckian effects on the quantum trajectories due to quartic and sextic corrections, the latter is much smaller than the former. We also calculate explicitly the finite vacuum energy density due to the fluctuations of the inflaton, and use the backreaction to constraint the value of parameters in the nonlinear dispersion relation. We have shown that during the early inflationary era when the Hubble parameter is close to the grand unification scale $h \sim 10^{15}$ GeV, the cosmological constant Λ decreases as (70) from well before horizon exit to well after horizon exit. Then, using the similar reduction mechanism, we propose a unified mechanism for generating a small cosmological constant through cascade transition in the history of the Universe.

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To establish that the cascade transition indeed works, it is necessary to have a much better understanding of the issues such as the specific scalar field potential models driving inflation at various phase transitions, the details of corresponding relaxation processes, and the experimental constraints on the parameters in nonlinear dispersion relations. Further work on these issues is in progress.

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AUTHORS PROFILE

Jung-Jeng Huang received M. S. degree from the Graduate Institute of Physics, National Taiwan Normal University, Taipei, Taiwan, in 1993. He is currently an Associate Professor in the Department of Mechanical Engineering, and the Director of the Committee of Physics Teaching Research, Ming Chi University of Technology, Taiwan. His current research interests include gravitation, cosmology, particle physics, and optoelectric physics.