Complex Effects in Dynamics of Prey-Predator Model with Holling Type II Functional Response

Vijaya Lakshmi. G. M, Vijaya S., Gunasekaran M.

Abstract— ***In this article, we study the discrete time prey-predator model by using Nicholson Bailey model (NB model) with Holling type II functional response. NB model with Holling type II is applied to know the Prey-predator dynamical system and investigated the fixed points and stability analysis. Graphs are drawn for different intrinsic growth rate to notice the effects of competitions for biologically reasonable range of parameter values. The stable existence of axial and interior fixed points of prey-predator is shown under different parameter values. Numerical simulations not only illustrate the results but also they exhibit the complex dynamic behaviours of the model.***

Keywords: Prey-predator system, Nicholson-Bailey model, Holling type II functional response, Stability analysis.

I. INTRODUCTION

In recent years the study of Holling type functional response in population dynamics has attracted very much attention and the qualitative analysis of predator prey systems with Holling type II or III functional response and prey refuge has been done by several papers [2,6,16,9]. The population dynamics of Prey-predator and Host parasitoid systems has been studied with Allee effect and without Allee effect [11,3,15,12,10]. The dynamical behaviour and Stability analysis of nonlinear discrete prey-predator and host parasite model has been studied and the discrete time host parasitoid model which are usually described by difference equation can produce much richer patterns than continuous time model [1,8,14,17]. It is well known that Nicholson Bailey model was developed in 1930`s to describe population dynamics of Host-parasite (prey-predator) system and it is one of the earliest realistic models of two species interaction was developed by Nicholson and Bailey, who applied it to the parasitoid Encarsia Formosa and the host Trialeurodes vaporariorum [5,13,14].

The prey-predator interaction has been described firstly by two pioneers Lotka(1924) and Volterra (1926) in two independent works. After them, more realistic prey-predator models were introduced by Holling suggesting three kinds of functional responses for different species to model the phenomena of predation [2]. The discrete prey-predator model to cover the Holling type II had a little attention in the discrete case till now, due to its complexities. Therefore the present work aims to analyse the dynamical complexities in a discrete-time prey-predator model with Holling type II functional response.

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That is, we shall focus our attention on analyzing how the Nicholson Bailey model with Holling type II response affects the dynamic complexities of prey-predator interactions. This paper is organized as follows: In section II, we formulated the discrete Nicholson Bailey model with Holling type II functional response. In section III the existence and stability of three fixed points are derived. In section VI numerical simulations are done for the analytic results, such as dynamics in a rectangular region (see Appendix). Finally, section VII draws the conclusion.

II. THE MODEL

We have the discrete generation, host-parasitoid Nicholson-Bailey model for two-dimensional system of difference equations

$$N_{t+1} = \lambda N_t e^{-ap_t}$$
$$P_{n+1} = cN_t\left(1-e^{-ap_t}\right) \quad (1)$$

Our analysis can be carried out most precisely by reference to an appropriate host-parasitoid system model. We shall use Holling type II functional response [16] to obtain bounded dynamics where the parameters discussed later.

$$N_{t+1} = \alpha \frac{N_t}{N_t+m} e^{-\beta p_t}$$
$$P_{t+1} = \gamma \frac{N_t}{N_t+m}\left(1-e^{-\beta p_t}\right) \quad (2)$$

Where α, β, γ, m are all positive parameters.

N_t *and* P_t are prey and predator population at time t.

N_{t+1} *and* P_{t+1} are prey and predator population size at time t+1 in terms of the population at time t. α is prey intrinsic growth parameter, β is the proportionality constant, γ is predator intrinsic growth parameter, m is half-saturation constant of predator [6] . $e^{-\beta p_t}$ is the probability that a prey escapes from predator and $\left(1-e^{-\beta p_t}\right)$ is probability that a prey attacked by predator.

In this paper, we study the dynamics of discrete Prey-predator model with Holling type II which has the following two different equations:

$$x_{t+1} = \alpha \frac{x_t}{x_t+m} e^{-\beta y_t}$$
$$y_{t+1} = \gamma \frac{x_t}{x_t+m}\left(1-e^{-\beta y_t}\right) \quad (3)$$

Where α, β, γ, m are defined in model (2). It is assumed that the initial value of solutions in system (2) satisfied

$x(0)>0, y(0)>0$ all the parameters are positive. It is easy to prove that if the initial value (x_0, y_0) is positive, then the corresponding solutions (x_t, y_t) is positive too.

III. FIXED POINTS AND LOCAL STABILITY

We now study the existence of fixed points of the system (3), particularly we are interested in the non-negative interior fixed point to begin and we list all possible fixed points.

(i) $E_0=(0,0)$ is trivial or extinction fixed point.

(ii) $E_1=(\alpha-m,0)$ is the axial or exclusion fixed point in the absence of the Predator (y = 0).

(ii) $E_2=(x^*,y^*)$ is the interior fixed point, where

$$x^*=\frac{\alpha\ln P}{\gamma\beta(P-1)} \text{ and } y^*=\frac{\ln P}{\beta} \qquad (4)$$

where $P=\frac{\alpha}{x+m}$ exists if and only if the following condition is satisfied

$B_1+\sqrt{B_1^2-4B_2^2}<2$ and $B_1-\sqrt{B_1^2-4B_2^2}<2$ where B_1 and B_2 are discussed in sec. V

IV. THE DYNAMICAL BEHAVIOUR OF THE MODEL

In this section, we investigate the local behavior of model (3) around each fixed point. The local stability analysis of the model (3) can be studied by computing the variation matrix corresponding to each fixed point. The variation matrix of the model at the state variable is given by

$$J(x_t,y_t)=\begin{bmatrix} \dfrac{\alpha m e^{-\beta y_t}}{(x_t+m)^2} & \dfrac{-\alpha\beta x_t e^{-\beta y_t}}{x_t+m} \\ \dfrac{\gamma m\left(1-e^{-\beta y_t}\right)}{(x_t+m)^2} & \dfrac{\gamma\beta x_t e^{-\beta y_t}}{x_t+m} \end{bmatrix} \qquad (5)$$

The characteristic equation of jacobian matrix be written as $\lambda^2-Tr+Det=0$ where Tr is the trace and Det is the determinant of the jacobian matrix $J=(x_t,y_t)$ which is defined as

$$Tr=\frac{\alpha m e^{-\beta y_t}}{(x_t+m)^2}+\frac{\gamma\beta x_t e^{-\beta y_t}}{x_t+m} \text{ and}$$

$$Det=\frac{\alpha\beta\gamma m x_t}{(x_t+m)^3}e^{-\beta y_t}$$

Hence the model (3) is a Dissipative dynamical system if $\left|\frac{\alpha\beta\gamma m x_t}{(x_t+m)^3}e^{-\beta y_t}\right|<1$ and Conservative dynamical one if and only if $\left|\frac{\alpha\beta\gamma m x_t}{(x_t+m)^3}e^{-\beta y_t}\right|=0$ and is a un dissipative dynamical system otherwise.

A. Remark:

Let $F(\lambda)=\lambda^2-B_1\lambda+B_2$ and if λ_1 and λ_2 are two roots of $F(\lambda)=0$ which are eigen values of fixed point (x,y). We recall some definitions of topological types for a fixed point (x,y). A fixed point (x,y) is locally asymptotically stable if $|\lambda_{1,2}|<1$.

Preposition 1: The fixed point E_0 asymptotically stable if $\alpha<m$ otherwise unstable fixed point.

Proof: In order to prove this result we estimate the eigen values of Jacobian matrix J at E_0. The Jacobian matrix for E_0 is $J_0=\begin{bmatrix} \frac{\alpha}{m} & 0 \\ 0 & 0\end{bmatrix}$.

Hence the eigen values of matrix are $\lambda_1=\frac{\alpha}{m}$ and $\lambda_2=0$,

By remark, if $\frac{\alpha}{m}<1$ which implies that $\alpha<m$ then E_0 is asymptotically stable otherwise unstable fixed point.

Preposition 2: The fixed point E_1 locally asymptotically stable if $0<m<\alpha \text{ and } 0<m<\frac{\alpha\beta+1}{\beta}$, otherwise unstable fixed point.

Proof: One can see that the Jacobian matrix for E_1 is given by $J_1=\begin{bmatrix} \frac{m}{\alpha} & -\beta(\alpha-m) \\ 0 & \beta(\alpha-m)\end{bmatrix}$.

Hence the eigen values of J_1 are $\lambda_1=\frac{m}{\alpha}$ and $\lambda_2=\beta(\alpha-m)$.

By remark, E_1 is stable if

$0<m<\alpha \text{ and } 0<m<\frac{\alpha\beta+1}{\beta}$ and un stable if

$m>\alpha \text{ and } m>\frac{\alpha\beta+1}{\beta}$.

B. Lemma:

If the eigen values of the Jacobian matrix of the fixed point are inside the unit circle of the complex plane, the fixed point of E is locally stable. Using Jury`s condition [4] we have necessary and sufficient condition for local stability of interior fixed point which are necessary and sufficient condition for $|\lambda_{1,2}|<1$.

(i) $1+Tr(J)+Det(J)>0$

(ii) $1-Tr(J)+Det(J)>0$

(iii) $|Det(J)|<1$

V. LOCAL STABILITY AND DYNAMICS BEHAVIOUR AROUND INTERIOR FIXED POINT E_2

We now investigate the local stability and bifurcation of interior fixed point E_2. The Jacobian matrix (5) at E_2 has of the form

$$J_2 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (6)$$

Where $a_{11} = \frac{\beta\gamma m(P-1)^2}{\alpha P(\log P)^2}$, $a_{12} = \frac{-\alpha^2\beta\log P}{P(\alpha\log P + m\beta(P-1))}$,

$a_{21} = \frac{\beta^2\gamma^3 m(P-1)^3}{\alpha^2 P(\log P)^2}$, $a_{22} = \frac{\alpha\beta\gamma\log P}{\alpha\log P + m\beta\gamma(P-1)}$,

where $P = \frac{\alpha}{x+m}$.

Its characteristic equation is

$$\lambda^2 - Tr + Det = 0 \quad (7)$$

where Tr is trace and Det is determinant of the jacobian matrix J_2 defines in eq. (6) Where $Tr = a_{11}+a_{22} = B_1$, $Det = a_{11}a_{22} - a_{12}a_{21} = B_2$.

By Lemma , using formulas of Tr and Det, the interior fixed point is locally stable if we find that

the inequality (i) is equivalent to $1-B_1+B_2>0$ which implies that $B_1 - B_2 < 1$, the inequality (ii) is equivalent to $B_1 + B_2 > -1$. the inequality (iii) is equivalent to $|B_2| < 1$.

For the interior fixed point E_2 the roots of eq.(7) are $\lambda_{1,2} = \frac{B_1 \pm \sqrt{B_1^2 - 4B_2^2}}{2}$. Both of the eigen values are local asymptotically stable if $B_1 + \sqrt{B_1^2 - 4B_2^2} < 2$ and $B_1 - \sqrt{B_1^2 - 4B_2^2} < 2$.

The eigen values in numerical simulations can be used to classify different types of dynamic behavior in a rectangular region.

VI. NUMERICAL SIMULATION

In this section we give the numerical simulations to verify our theoretical results proved in the previous section by using MATLAB programming. We also confirm the results by visual representation of the system for some values of parameters. We provide some numerical evidence for the qualitative dynamic behavior of the map (3). We use mathematical functions over a rectangular region to illustrating the above analytic results and dynamic behavior of map (3) as the parameters varying. The diagrams are considered in three cases which was explained in the above prepositions.

Fig-(1) (for figures see Appendix) is surface diagram in a rectangular region for ∝=2, a stable coexistence is noticed when ∝ < β (at extinction point). **Fig-(2)** is a surface diagram in rectangular region when ∝=7, m<∝ and when predation goes to extinction rate, stable coexistence is noticed. **Fig-(3)** is a surface diagram in rectangular region when ∝=7 at the interior fixed point and the absolute value is less than 1. Hence by remark the equilibrium point is local asymptotically stable. **Fig-(4)** is a surface diagram in rectangular region when ∝=35 at the interior fixed point and the absolute value is greater than one, hence it is a unstable coexistence is noticed. We conclude that the numerical simulations agree with the analytical results on NB model with Holling type II functional response.

VII. CONCLUSION

In this paper, we analyzed dynamics of a nonlinear discrete-time prey-predator system. This paper presents some innovative analysis with respect to previous studies on stability analysis in prey predator system with Holling type II functional response [2,6,16,9]. The study of host-parasitoid with the help of modified N-B model yields interesting results. In general parasitoid lives in or on the host and necessarily kills the host, similarly predators kills its prey. In the present study we discussed about the local stability of prey predator by using modified NB model with Holling type II functional response. We have introduced certain new parameters to discrete-time prey predator model and obtain equilibrium points. The numerical solution of the population size shows a succession of stable dynamics. We also showed that the system exhibits a huge variety of complicated dynamical behaviour in a rectangular region.

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AUTHORS PROFILE

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APPENDIX

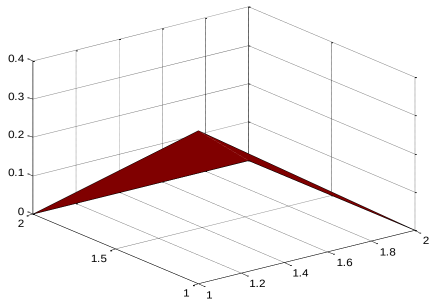

Fig-(1)

Prey predator dynamics at extinction point E_0

∝=2, γ =1, β=0.5, m=5

$\lambda_1 = 0.4$ and $\lambda_2 = 0$.

One of the eigen value $|\lambda| < 1$, E_0 is stable.

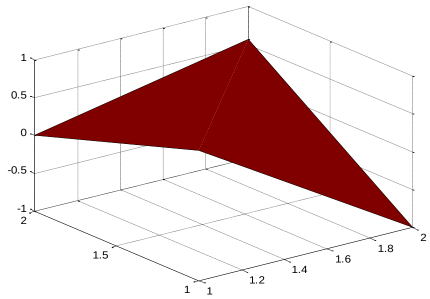

Fig-(2)

Prey predator dynamics at axial fixed point E_1

∝=7, γ =4, β=0.5, m=5

$|\lambda_1| = 0.7143$ and $|\lambda_2| = 0.5714$

Since $|\lambda_{1,2}| < 1$, E_1 is Local asymptotically stable

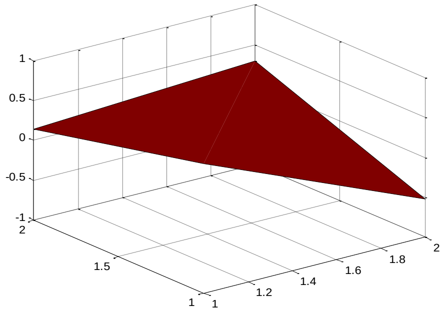

Fig-(3)

Prey predator dynamics at interior fixed point E_2

∝=7, γ =4, β=0.5, m=5

$\lambda = 0.4622 \pm 0.2166$ i

Since $|\lambda| = 0.5195 < 1$, E_2 is Local asymptotically stable.

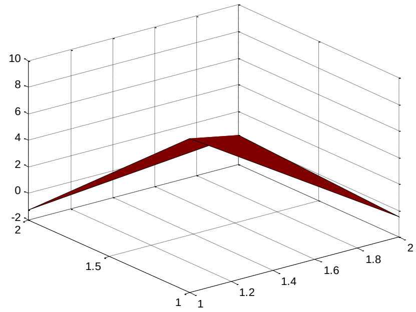

Fig-(4)

Prey predator dynamics at interior fixed point E_2

∝=35, γ =15, β=0.5, m=5

$|\lambda_1| = 9.7392$ and $|\lambda_2| = 0.1384$

Since $|\lambda_1| > 1$, E_2 is un stable.

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