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Dynamics in Discrete Time Prey-Predator System with Quadratic Harvesting on Prey

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Abstract*— This paper describes the Stability Analysis of Discrete time Prey-Predator on equilibrium and find the local Stability conditions near equilibrium points. A geometrical representation of the trajectories of dynamical system in the phase portraits are obtained for different set of parameter and time series for selective range of growth parameter are represented here. Harvesting activity of the Prey and Prey-Predator population are investigated through Chaotic Dynamic System. Times Series for both Prey and Predator separately analyzed for different values of harvesting.. Numerical Simulations are presented here for explaining complex dynamical behaviors of Bifurcation

***Keywords* — Prey-Predator system, Local Stability, Quadratic harvesting, Phase portraits**

I. INTRODUCTION

Predator –Prey models are building blocks of the Bio-Eco systems as Biomasses are grown out of their resource masses. Species compete, evolve and scatter simply for the purpose of seeking resources to sustain their struggle for their very existence. Depending on their specific settings of applications, they can take the forms of resource-consumer, plant-herbivore, parasite-host etc. There are many instances in nature where one species of animal feeds on another species of animal, which in turn feeds on other things. The first species is called the Predator and the second is called the Prey. What actually happens in nature is that a cycle develops where at some time the prey may be abundant and the predators few. Because of the abundance of prey, the predator population grows and reduces the population of prey.

An important problem of *Ecology* , the science which studies the interrelationships of organisms and their environment, is to investigate the question of coexistence of the two species. To this end, it is natural to seek a mathematical formulation of this predator-prey problem. and to use it to forecast the behavior of populations of various species at different times.

The differential equations are very much helpful in many areas of science. The Lotka-Volterra model is composed of a pair of differential equations that describe predator-prey (or herbivore-plant, or parasitoid-host) dynamics in their simplest case (one predator population, one prey population).

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It was developed independently by. Alfred Lotka[11] and Vito Volterra[13] in the 1920's, and is characterized by oscillations in the population size of both predator and prey, with the peak of the predator's oscillation lagging slightly behind the peak of the prey's oscillation. The model makes several simplifying assumptions:

- The prey population will grow exponentially when the predator is absent;
- The predator population will starve in the absence of the prey population (as opposed to switching to another type of prey);
- Predators can consume infinite quantities of prey;
- There is no environmental complexity.

After them more realistic Prey-Predator model was introduced by Holling [5] in 1965, which dealt many kinds of Prey-Predator model in Ecology. M.Danca.et.al [2] gave analytical model of competition between two dimensional map and rich Dynamics. J.Dhar[3] proposed a mathematical model to study role of supplementary self-renewable resource on population in two-patch habitat. They studied the dynamics of corresponding discrete models obtained by Euler method in Jing et al[6], and Jing andYang[7] . Also complex behavior of predator-prey system obtained by Euler method examined in Liu andXiao[10]. Chaotic dynamics of a discrete prey-predator model with Holling type II studied in Agiza et al[1]. N.P Kumar[8][9] et al., studied the mathematical model of commensalism between two species with limited sources. Also [12] studied a prey predator model in which the predator is provided with alternative food in addition to the prey and the prey predator harvested proportional to the population size.

II. MATHEMATICAL MODEL

The model is

$$\frac{dx}{dt} = rx(1-x) - axy - hx^2$$
$$\frac{dy}{dt} = -cy + bxy \qquad \text{----(1)}$$

Where $x(t)$, $y(t)$ be the population densities of prey and predators, r represents natural grow of Prey in the absence of Predator. 'a' represents effect of predation on prey.'c' represents natural death rate of Predator in the absence of prey.'b' represents efficiency and propagation rate of predator in the presence of prey , 'h' harvesting effect. It is assumed that initial value of the system (1) satisfied with $x(0) > 0, y(0) > 0$ and all parameters are positive.

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III. FIXED POINT AND LOCAL STABILITY

We now study the existence of fixed point of the system (1) particularly we are interested non-negative or interior fixed point. To begin with we list all possible fixed points.

(i). $E_0 = (0, 0)$ is trivial point.

(ii). $E_1 = (\frac{r}{r+h}, 0)$ in the absence of predator , y =0.

(iii). The interior fixed point is $E_2(x^*, y^*)$ where

$$x^* = \frac{c}{b} \quad y^* = \frac{br - c(r+h)}{ab} \qquad \text{---------- (2)}$$

IV. DYNAMIC BEHAVIOR OF THE MODEL

In this subsection, we investigate the local behavior of the model (1) around each fixed point. The local stability analysis of the model (1) can be studied by computing the variation matrix corresponding to each fixed point. The variation matrix of the model at state variable is given by

$$J(x,y) = \begin{pmatrix} r(1-2x) - ay - 2hx & -ax \\ by & bx - c \end{pmatrix}$$

The determinant of the Jacobian $J(x,y)$ is $Det = [(r(1-2x) - ay - 2hx)(bx-c) + axby]$ Hence the model (3) is dissipative dynamical system when $|[(r(1-2x) - ay - 2hx)(bx-c) + axby]| < 1$ .In the following Lemma is useful in the study of nature of fixed points.

Non-linear systems are much harder to analyze since in most cases they do not possesses quantitative solution even when explicit solution are available they are often too complicated to provide much insight. One of the most useful techniques for analyzing non-linear system quantitavely is the linearised stability technique the stability of the system is investigated by obtaining Eignvalues of the Jacobian matrix is associated with fixed points[1],[4] in order to study the stability of the fixed point model we first give the following theorem:

Theorem Let $p(\lambda) = \lambda^3 + B\lambda^2 + C\lambda + D$ be the roots of $p(\lambda) = 0$.Then the following statements are true

a) If every root of the equation has absolute value less than one, then the fixed point of the System is locally asymptotically stable and fixed point is called a sink.
b) If at least one of the roots of equation has absolute value greater than one then the fixed point of the system is unstable and fixed point is called saddle.
c) If every root of the equation has absolute value greater than one then the system is a source.
d) The fixed point of the system is called hyperbolic if no root of the equation has absolute value equal to one. If there exists a root of equation with absolute value equal to one then the fixed point is called non-hyperbolic.

PREPOSITION: 1 The fixed point E_0 of the system is locally asymptotically stable if $r, c < 1$ otherwise unstable fixed point.

PROOF: In order to prove the result, we estimate the eigenvalue of Jacobian matrix J at E_0.

The Jacobian matrix for E_0 is

$$J(E_0) = \begin{pmatrix} r & 0 \\ 0 & -c \end{pmatrix}$$

Hence the Eigen values of $J(E_0)$ are $\lambda_1 = r, \lambda_2 = -c$

Thus for the values of $r, c < 1$ the equilibrium point E_0 is locally asymptotically stable otherwise E_0 is unstable fixed point..

PREPOSITION: 2 The fixed point E_1 of the system is locally asymptotically stable if $r < 1$ and $br < c(r+h)$ otherwise unstable fixed point.

PROOF: By linearizing system (1) at E_1 we obtain the Jacobian

The Jacobian matrix for E_1 is

$$J(E_1) = \begin{pmatrix} -r & \frac{-ar}{r+h} \\ 0 & \frac{br}{r+h} - c \end{pmatrix}$$

The characteristic equation of $J(E_1)$ is

$$\lambda^2 - trJ(E_1) + DetJ(E_1) = 0$$

Where $trJ(E_1) = \frac{br - (r+c)(r+h)}{r+h}$

and $DetJ(E_1) = \frac{r^2(c-b) + hcr}{r+h}$

Solving we get the Eigen values

$$\lambda_1 = -r, \lambda_2 = \frac{br}{r+h} - c$$

It is clear that equilibrium point is sink if $r < 1$ and $br < c(r+h)$ it shows it is locally asymptotically stable .Also if $r > 1$ and $br > c(r+h)$ is unstable and fixed point is called saddle.

LEMMA(1): If the Eigen values of the Jacobian matrix of the fixed point are inside the unit circle of the complex plane, the fixed point of E is locally stable. Using Jury`s condition we have necessary and sufficient condition for local stability of interior fixed point which are necessary and sufficient condition for $|\lambda_{1,2}| < 1$.

(i) $1 + Tr(J) + Det(J) > 0$

(ii) $1 - Tr(J) + Det(J) > 0$

(iii) $|Det(J)| < 1$

PREPOSITION: 3 The interior equilibrium point $E_2(x^*, y^*)$ of the system (1) is locally stable if $h > r$ and $br > c(r+h)$ where x^* , y^* is given by the equation (2)

Proof: By linearizing system (1) at E_1 we obtain the jacobian

The Jacobian matrix for E_2 is

$$J(E_2)=\begin{pmatrix} \dfrac{-c(r+h)}{b} & \dfrac{-ac}{b} \\ \dfrac{rb-c(r+h)}{a} & 0 \end{pmatrix}$$

The characteristic equation of $J(E_2)$ is

$$\lambda^2 - trJ(E_2) + DetJ(E_2) = 0 --- (3)$$

Where $trJ(E_2) = B_1 = \dfrac{-c(r+h)}{b}$ ------ (4)

and $DetJ(E_2) = B_2 = \dfrac{c(c(h+r)-br)}{b}$ ----(5)

By Lemma (1), using formulas of Tr and Det, we find that inequality (i) is equivalent to $1 - B_1 + B_2 > 0$ which implies that $B_1 - B_2 < 1$.

The second inequality (ii) is equivalent to $B_1 + B_2 > -1$.

The third inequality (iii) is equivalent to $|B_2| < 1$.

By solving the characterstic equation (3) at the interior fixed point E_2 the roots (EigneValues at E_2) will be

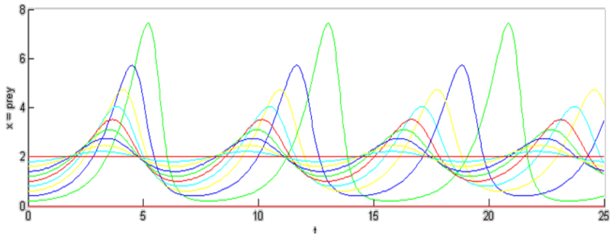
$$\lambda_{1,2} = \frac{Tr(J(E_2)) \pm \sqrt{(Tr(J(E_2)2 - 4DetJ(E_2))}}{2}.$$

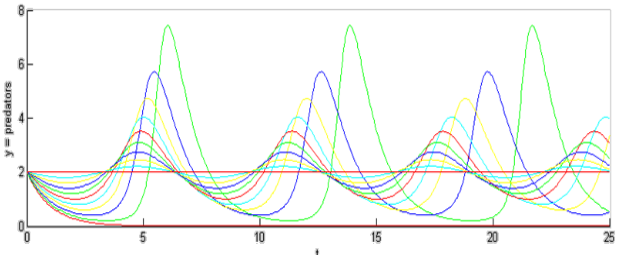
Where $Tr(J(E_2))$ and $DetJ(E_2)$ given by equations (4) and (5). The Eigen Values in numerical can be used to classifiy different type of bifurcation.

5. Numerical simulation

In this section we give the numerical simulations to verify our theoretical results proved in the previous section by using MATLAB programming. We also confirm the results by visual representation of the system for some values of parameters. We provide some numerical evidence for the qualitative dynamic behavior of the map (1).

Following Diagram illustrates Time Series Plots for both Prey –Predator and Phase Portraits for Different values of r,h,a,b and c





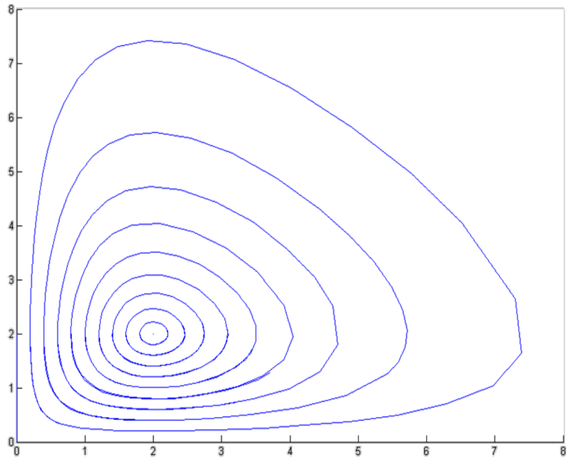


Figure 1 TIME SERIES FOR PREY –PREDATOR AND PHASE PORTRAIT

$r = 1, a = 0.5, b = 0.5, c = 0.5, h = 0.25$

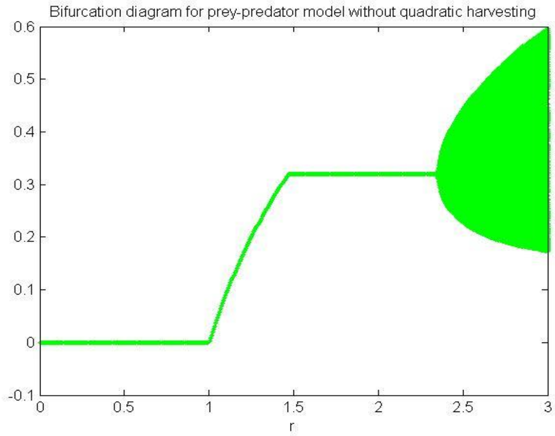


Figure2 BIFURCATION FOR WITHOUT QUADRATIC HARVESTING

$r = 0to4, a = 2, b = 4, c = 0.5, h = 0$

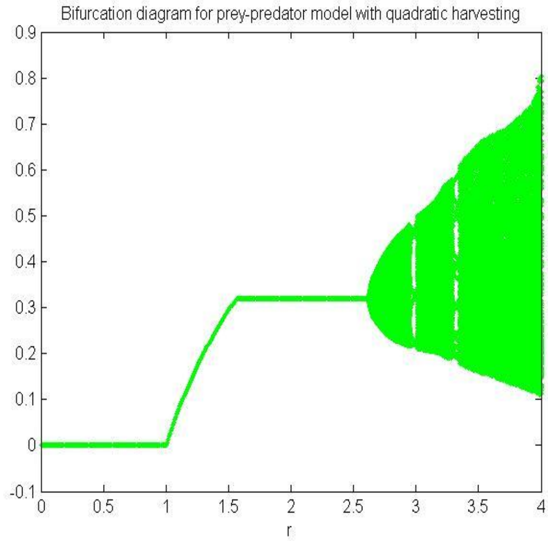


Figure 3 BIFURCATION FOR WITH QUADRATIC HARVESTING

$r = 0to4, a = 2, b = 4, c = 0.5, h = 0.5$

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V. CONCLUSION

This paper investigated the stability of quadratic harvesting equilibrium model and conditions for stability are obtained. We conclude that equilibrium of prey-predator model (1) is stable for harvesting activity and when we consider the harvesting activity of prey, the population size of predator decreased then the system will be unstable. The purpose of the work is to give the mathematical analysis of the model and to discuss some significant results are expected from the biological forces. We presented numerical simulations to show the dynamical behavior of the system which is being harvested. Also we exhibit Bifurcation diagram for Prey-Predator model with and without quadratic harvesting for different values.

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