

Clutter Removal for RADAR Wind Profiler using Wavelet Thresholding

M. Krupa Swaroopa Rani, G. Kiran Kumar, M. Krishnaiah, K. Kameswara Rao

Abstract--Atmospheric Signal processing has been one field of signal processing where there is a lot of scope for development of new and efficient tools for cleaning of the spectrum, detection and estimation of the desired parameters. The field of digital signal processing is a very active area for research and applications. Atmospheric signal processing deals with the processing of the signals received from the atmosphere when manually stimulated using atmospheric Radar. Removal of clutter in the radar wind profiler is the utmost important consideration in radar. In this paper, we implement wavelet thresholding for removing clutter from wind profiler Radar data. By applying the concept of discrete multi-resolution analysis and non-parametric estimation theory, we develop wavelet domain thresholding rules, which identifies the coefficients relevant for clutter and suppresses them and increases the accuracy of wind vector reconstruction.

Index Terms— Clutter, Signal Processing, Wind Profiler, Wavelet Thresholding.

I. INTRODUCTION

RADAR (Radio Detection and Ranging) is a device that sends out electromagnetic waves. These waves reflect off of objects in space, and a proportion of the original wave energy is actually bounced back towards the RADAR. The RADAR then reads this returning signal and analyzes it. This returning signal can be processed to determine many properties about the original object that the wave reflected off of. Two examples that can be determined from the returned signal are the location of the object (distance away from the radar itself) as well as the velocity of the object in relation to the radar.

II. CONCEPT OF RADARS

Radar itself is an abbreviation for Radio Detection and Ranging. Radar systems send out modulated waveforms using antennas in order to transmit electromagnetic energy into a specific volume of space to search for targets. Objects (i.e. targets) within a certain volume will reflect part of the energy (radar returns or echoes) back to the radar. From these radar returns, the radar receiver then extracts information such as velocity and range, angular position, and other identifying characteristics. If relative motion exists between target and radar, the shift in the carrier frequency of the reflected wave (Doppler effect) is a measure of target's

relative (radial) velocity and may be used to distinguish moving targets from stationary objects. National Atmospheric Research Laboratory (NARL) at Gadanki (13.47°N, 79.18°E) near Tirupati, India has been operating a 1280 MHz atmospheric radar for studying structure and dynamics of lower atmosphere.

The operating frequency of LAWP is 1280MHz. The phased antenna array consists of 8x8 elements occupying an area of 1.4mx1.4m. It transmits a peak power of 0.8KW. The number of coherent integrations can be in the range of 4-1000. The number of Fast Fourier Transform (FFT) points can be from 1-256. To obtain the wind speed and direction, LAWP measures data in three directions, namely zenith, north and east in one observation cycle. The typical height coverage in the clear air is 3-4Km and 10Km during precipitation. The selected parameters of LAWP are shown in table.

Specifications

□ Frequency	1280 MHz
□ Technique	Doppler beam Swinging
□ Antenna Type	Microstrip Patch Array
□ Array Size	8x8 (1.4 m x 1.4 m)
□ Beam Width	9°
□ Beam Former	Passive
□ Beams	25
□ Tx/Rx Type	Solid State Transceivers
□ Peak Power	0.8 kW
□ Duty Ratio	Upto 10%
□ Pulse Width	0.25 – 8.0 μs
□ NCI	4 - 1000
□ NFFT	32 - 1024
□ Range Bins	1-256
□ Receiver	Super Hetrodyne
□ Detection	Direct IF Digital
□ Dynamic Range	70 dB
□ Min.Height	100 m
□ Max.Height	3-5 km

Most of these RWP employ the Doppler-beam swinging (DBS) method for the determination of the vertical profile of the horizontal wind and, under certain conditions, the vertical wind component. These radars transmit short electromagnetic pulses in a fixed beam direction and sample the small fraction of the electromagnetic field backscattered to the antenna.

At least three linear independent beam directions are required to transform the measured 'line-of-sight' radial velocities into the wind vector. Due to the nature of the acting atmospheric scattering processes, the



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received signal is several orders of magnitude weaker than the transmitted signal. The received signal is Doppler shifted, which is used to determine the velocity component of “the atmosphere” projected onto the beam direction. The goals of signal processing, as summarized by [1], are:

- to provide accurate, unbiased estimates of the characteristics of the desired atmospheric echoes;
- to estimate the confidence/accuracy of the measurement;
- to mitigate effects of interfering signals;
- to reduce the data rate.

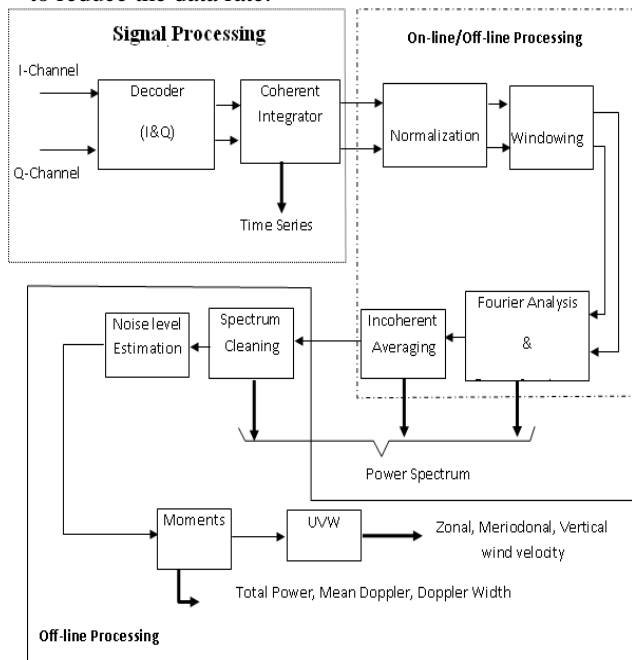


Fig 1. Data Processing Steps of Radar Wind Profiler.

Digital signal processing in a system using an analog receiver starts with the sampling of the in- and quadrature phase components of the received signal at a rate that is determined by the pulse repetition period T. To reduce the data rate for further processing, hardware adder circuits perform a so-called coherent integration [2][3], adding some N (typically ten to hundred) complex samples together. If the radar system uses pulse compression techniques (e.g. phase coding using complementary sequences), then the next step is decoding [4][5][6]. The coherently averaged and decoded samples are then used to compute the Doppler spectrum using the Windowed Fourier Transform (FFT) and the Periodogram method [1]. In our system, a Fourier transformed Hanning-window is convolved with the result of the FFT. A number (typically some ten) of individual Doppler spectra is then incoherently averaged to improve the detectability of the signal [7]. Finally, the noise level is estimated with the method proposed by [2], and the moments of the maximum signal in the spectrum are computed over the range where the signal is above the noise level [8]. The problem with this type of signal processing is the underlying assumption that the signal consists of only two parts: the signal, that is produced by one atmospheric scattering process, and noise (different sources, mainly thermal electronic noise and cosmic noise). This is certainly not true, especially at UHF, where the desired atmospheric signal itself is often the result of two distinct scattering processes, namely scattering at inhomogeneities of the refractive index

(Bragg scattering) and scattering at particles, such as droplets or ice crystals (Rayleigh scattering) (see, for instance, [9] [10] [11] [12]). Therefore, even the desired atmospheric signal may have different characteristics. But, as experience shows us, the most serious problems are caused by the following contributions to the signal:

Ground Clutter. Echo returns from the ground surrounding the site, which emerge from antenna’s side lobes.

Intermittent Clutter. Returns from unwanted targets, such as airplanes or birds, from both the antenna’s main lobe and the side lobes.

III. APPLYING MULTIREOLUTION ANALYSIS AND STATISTICAL ESTIMATIONS

For the problem at hand, the goal of the signal processing should be signal component separation, i.e. an automatic, reliable and stable extraction of the different contributions to the signal (noise, clutter, interference). Motivated by [13] [14] [15] [16] and [17], our purpose was to embed the filtering procedure into the known mathematical theory of wavelets. In general, mathematical experience concerning problems related to contamination removal or denoising shows that usually more than time domain filtering and Fourier domain filtering techniques are required to obtain optimum results. Often, most of the existing and implemented methods are insufficient. The main reasons for the particular effectiveness of wavelet analysis can be summarized as follows:

- The fact that contamination appears often instationary or transient, and with a priori unknown scale structure, favors the superior localization properties of the wavelets. A wavelet expansion may allow the separation of signal components that overlap both in time and frequency [18].
- In order to effectively localize clutter components, one can use a great variety of wavelet filters [13] [19] [20]. To choose a certain wavelet that especially suits the desired signal component, one can determine the properties of the clutter signal; otherwise, one can select a wavelet empirically.
- The wavelet expansion coefficients, β_{jk} , drop off rapidly for a large class of signals, which makes the expansion very efficient [18].
- The fast wavelet transform has a computationally complexity that is lesser than or equal to the fast Fourier transform; the algorithm is recursive [21] [15] [18]. This allows for an efficient implementation on digital computers. Thus, the application of wavelet techniques to our particular problem seems to be promising. Before we start, let us briefly repeat the basics of multi-resolution analysis. Let $L_2(\mathbb{R})$ be the space of functions of finite energy. Let ϕ be some function in $L_2(\mathbb{R})$, such that the family of translates of ϕ form an orthonormal system. We define $\phi_{jk}(x) = 2^{j/2} \phi(2^j x - k)$, $j \in \mathbb{Z}, k \in \mathbb{Z}$. (1)
- Assuming that ϕ is chosen in such a way that the spaces are nested: $\bigcup_{j \geq 0} V_j$ and that $V_j \subset V_{j+1}$, $j \in \mathbb{Z}$ is dense in $L_2(\mathbb{R})$
- Further, we define linear spaces by $V_0 = \{f(x) = \sum_k c_k \phi(x - k) : \sum_k |c_k|^2 < \infty\}$. (2)
- $V_j = \{h(x) = f(2^j x) : f \in V_0\}$, $j \in \mathbb{Z}$ (3)
- then the sequence $\{V_j, j \in \mathbb{Z}\}$ is called a multi-resolution analysis. ϕ is called the father wavelet. Furthermore, one may define subspaces W_j by



$$V_{j+1} = V_j \oplus W_j \quad (4)$$

And iterating this we have $\cup V_j = V_0 \oplus \oplus_j W_j$ and $L_2(R) = V_0 \oplus \oplus_j W_j$. (5)

Assuming that our data may be described by some $f \in L_2(R)$ we can represent the signal as a series $f(x) = \sum_k \alpha_k \phi_{0k}(x) + \sum_j \sum_k \beta_{jk} \psi_{jk}(x)$ (6)

Where $\{\psi_{jk}\}, k \in \mathbb{Z}$ is an orthonormal basis in W_j . The function ψ is called mother wavelet. This expansion is a special kind of orthogonal series. Hence, it would be useful to search in the framework of nonparametric statistical estimation theory for an applicable method to solve our problem [7]. In case of orthogonal series estimation, the idea of reconstructing the desired atmospheric signal is simple. Basically, we replace the unknown wavelet coefficients in the wavelet expansion by estimates which are based on observed data. For that, we need a selection procedure to choose relevant coefficients since the main emphasis of performing wavelet domain filtering is to create a suitable, i.e. problem matched, coefficient selecting procedure. To separate the atmospheric signal component, we apply statistical estimation theory. A side effect of using statistics is to obtain a measure of reconstruction quality. A typical quality measure is a loss function/ estimation error. Minimizing the error function reveals an objective evaluation and a self-acting filter algorithm. The following sub-section describes the construction of our atmospheric-signal-estimator. In advance, we briefly remark that in the following section, we assume that our signal belongs to some Besov space, i.e. a generalized mathematical function space. One special example is the previously introduced function space $L_2(\mathbb{R})$. But sometimes it makes more sense to suppose that the derivatives of our signal are of finite energy as well. In this and other situations, the framework of Besov spaces is an adequate mathematical tool for our application. A Besov space, denoted by B_{pq}^s , depends on three parameters: s smoothness, the number of bounded derivatives and p, q which describe the underlying function space $L_q(L_p)$. In the following, we make use of some well-known facts of estimation theory, which are valid for almost all Besov spaces [22] [23] [24] [25]. If our signal is an element of one of these spaces (which is true for all practical signals), we can adapt wavelet threshold estimators. The main advantage of this framework is that we can use existing rules for evaluating bounds and rates of convergence for our loss function, which describes the quality of our reconstructed atmospheric signal component. By optimizing bounds and rates of convergence, we obtain self acting algorithms. For our purpose, we only need the following characterization of Besov spaces: A function f belongs to B_{pq}^s if

$$J_{pq}^s(f) = \|\alpha\|_{l_p} + (\sum_{j \geq 0} (2^{j(s+1/2-1/p)} \|\beta_j\|_{l_p})^q)^{1/q} < \infty \quad (7)$$

For our calculations, we assume that the function is in $L_2(\mathbb{R})$ and s is small.

From given measurements (Y_1, \dots, Y_n) , we want to estimate the function f in the simple model.

$$Y_i = f(X_i) + \varepsilon_i \quad (8)$$

We assume that we have the X_i on a regular grid and ε is a random variable (a stochastic process which describes all non-atmospheric components). The basic idea is to replace the wavelet coefficients in the series expansion by empirical estimates.

$$\hat{\alpha}_k = \frac{1}{n} \sum_{i=1}^n Y_i \phi_{0k}(X_i) \text{ and } \hat{\beta}_{jk} = \frac{1}{n} \sum_{i=1}^n Y_i \psi_{jk}(X_i) \quad (9)$$

where the X_i are time stamps and the Y_i are observations. A straightforward linear estimation is given by the projection onto a subspace V_{j_1}

$$\hat{f}_{j_1}(x) = \sum_k \hat{\alpha}_k \phi_{0k}(x) + \sum_{j=0}^{j_1} \sum_k \hat{\beta}_{jk} \psi_{jk}(x) \quad (10)$$

Obviously, this kind of linear estimation includes oscillating components, in particular, the clutter components. This phenomenon occurs because we have taken the whole set of wavelet coefficients up to scale j_1 , i.e. we have not performed any filtering step thus far. In the following, we need a suitable selection procedure for the coefficients in order to perform the necessary filtering step. We apply a so-called hard thresholding and soft thresholding, respectively. This methodology was introduced and adapted to several problems by [7] [8]. It is based on taking the discrete wavelet transform (using a multiresolution analysis), passing the transform through a threshold (actually, the expansion coefficients are thresholded) and then taking the inverse DWT to obtain a filtered reconstruction. The functions for hard and soft thresholding are defined by

$$\theta^h(u) = \begin{cases} u, & |u| \geq \lambda \\ 0, & |u| < \lambda \end{cases} \quad (11)$$

$$\text{and } \theta^s(u) = \begin{cases} \left(u - \frac{\lambda u}{|u|}\right), & |u| \geq \lambda \\ 0, & |u| < \lambda \end{cases} \quad (12)$$

The modified functions for hard and soft thresholding are defined by

$$\eta^h(u) = \begin{cases} u, & |u| < \lambda \\ 0, & |u| \geq \lambda \end{cases} \quad (13)$$

$$\text{and } \eta^s(u) = \begin{cases} u, & |u| < \lambda \\ \lambda u/|u|, & |u| \geq \lambda \end{cases} \quad (14)$$

Here, λ is an adequate threshold. Applying this rule to our linear wavelet estimator, we obtain a nonlinear estimator

$$f^*(x) = \sum_k \eta^*(\hat{\alpha}_k) \phi_{0k}(x) + \sum_{j=0}^{j_1} \sum_k \eta^*(\hat{\beta}_{jk}) \psi_{jk}(x) \quad (15)$$

where η^* is η^s or η^h , respectively.

If the threshold λ is specified according to the asymptotic distribution of the empirical coefficients, then only those coefficients remain which are supposed to carry significant signal information. These are finally used for the reconstruction by the inverse wavelet transform. The resulting non-linear estimator does not only provide local smoothers, but, in many situations, achieves the nearminimax L_2 -rate for the risk of estimation, i.e. for (random) thresholds λ_{jk} satisfying

$$\sigma_{jk} \sqrt{2 \log M_j} \leq \lambda_{jk} \leq C \sqrt{\frac{\log n}{n}} \quad (16)$$

for any positive constant C,

where σ_{jk} is the variance and M_j denotes the number of the coefficients used in the nonlinear estimator. The optimal threshold rate $(\frac{1}{n})^{\frac{2s}{2s+1}}$ is attained only for the ideal threshold. However, in practice, this is unknown. Therefore, we have to replace σ_{jk} by some estimation $w_{\hat{\sigma}_{jk}}$ which results in random thresholds $\hat{\lambda}_{jk} = \hat{\sigma}_{jk} \sqrt{2 \log M_j}$. Hence, the log term has to be understood as the price for some data-driven threshold rule, and it originates due to the estimation of the unknown variance $\sigma_{jk}^2 = \text{Var}(\hat{\beta}_{jk})$

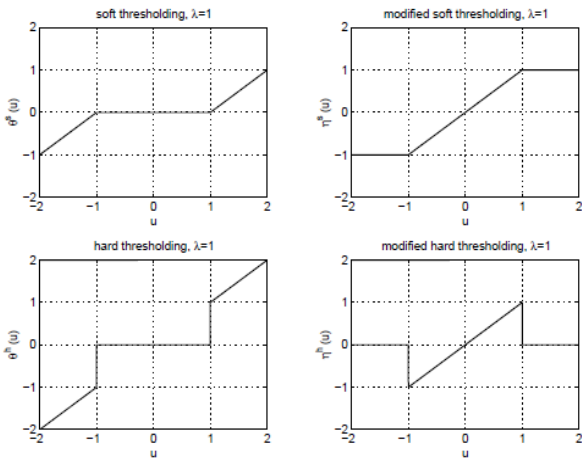


Fig 2. This figure shows hard and soft thresholding.

IV. WAVELET THRESHOLDING

In this section, we will demonstrate the performance of nonlinear wavelet filtering. For a better understanding, we particularize Fig. 1 to see where we have inserted the wavelet tool. To apply our procedure, a more substantiated algorithm flow diagram is shown in the figure below. Following the first box in the algorithm flow diagram, one has first to determine the analyzing wavelet (high and low pass filter coefficients). Usually, the decomposition of a signal in a basis (i.e. a wavelet series) has the goal of highlighting particular properties of the signal.

There have been no detailed investigations thus far about the regularity properties of contaminating wind profiler signals, but there is evidence that these can be both “quite regular” (ground clutter) or “not so regular” (intermittent clutter). Thus, the Daubechies family was selected. The order of the Daubechies wavelet was chosen according to the regularity condition, which we have conservatively chosen to be rather small ($s \leq 1$). To approximate correctly a function of B^s_{pq} , we need to select an analyzing wavelet of regularity $[s] + 1$. A wavelet with regularity of the order of $s = 2$ and minimal In the problem of wind profiler signal filtering, the desired atmospheric signal component can be contaminated with spurious signal components. The ultimate goal is obviously to find a wavelet basis, which would allow a separation of the desired and the unwanted parts of the signal, i.e. which would have the ability to approximate the unwanted signal components (ground clutter, intermittent clutter) with only a few non-zero wavelet coefficients. In other words, the wavelet ψ has to be chosen in such a way that a maximum number of wavelet coefficients, β_{jk} are close to zero. This depends primarily on the regularity of the (contaminating) signal f , the number of vanishing moments of the wavelet ψ , and the size of the wavelets support. If f is regular and has ψ enough vanishing moments, then the coefficients β_{jk} are guaranteed to be small for small scales. If, however, the signal f contains isolated singularities, the strategy to have a maximum number of small wavelet coefficients would be to reduce the support size of the wavelet. Unfortunately, there is a tradeoff between both properties for orthogonal wavelets: if ψ has p vanishing moments, then its support size is at least $2p - 1$. The best compromise between those two requirements are Daubechies wavelets, which are optimal in the sense that they have minimum support for a given number of vanishing moments. compact support is the

Daubechies-2-wavelet; hence, we have chosen this one for our calculations.

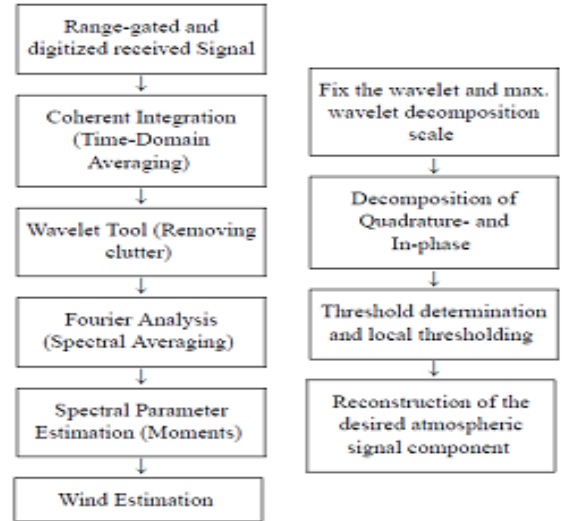
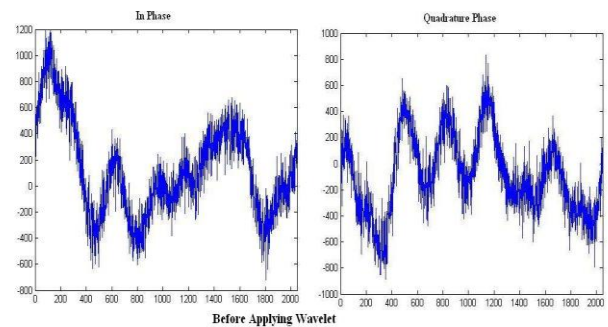
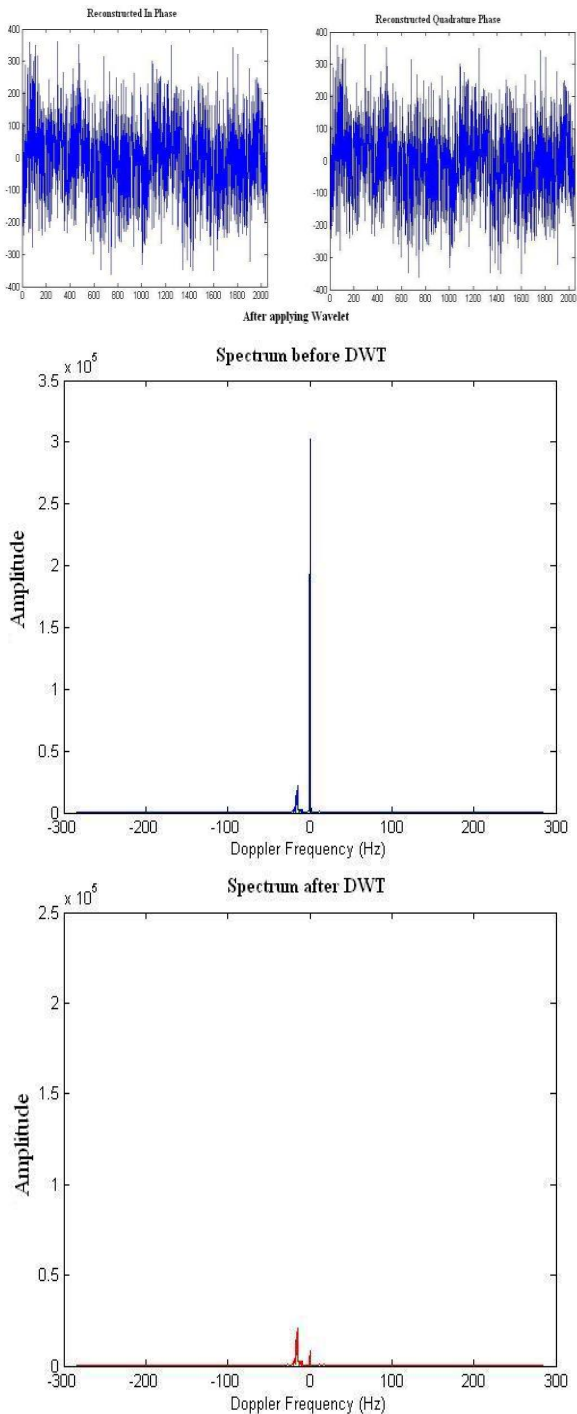


Fig 3. Left: The flow diagram extended by the wavelet tool. Right: The wavelet algorithm flow diagram.

Mathematically, it is no problem to increase the wavelet order (regularity), but the wavelet support size and the number of filter coefficients also increases, and this will decelerate the algorithm. Finally, we note, in passing, that we have concentrated on the fast wavelet transform (multiresolution analysis), which is a special case of the discrete wavelet transform. Obviously, for an online algorithm, the number of operations per data point is limited. The fast wavelet transform is, therefore, the best choice, since it has the highest numerical efficiency (i.e. it is faster than the fast Fourier transform). This, of course, restricts the possible choices of the underlying basis wavelet. The number of decomposition scales is determined by balancing the stochastic and the deterministic part of the MISE. Thus, the optimal scale may be evaluated automatically by the rule $2^{j1(n)} \cong n^{\frac{1}{(2s+1)}}$. After fixing the main parameters, one may start the wavelet decomposition of the in-phase and the quadrature-phase time series. To separate the atmospheric component, the algorithm calculates for each decomposition level the local thresholds λ_{jk} .

V. RESULTS





VI. CONCLUSION

This paper discusses an signal processing algorithm which implements discrete multiresolution analysis and nonlinear estimation theory for separating the atmospheric Doppler signal in RWP measurements in the presence of contaminating signals. We have demonstrated that wavelet thresholding is effective in removing ground and intermittent clutter (airplane echoes) from the RWP raw data (I/Q timeseries).

REFERENCES

1. Keeler, R. J. and Passarelli, R. E., Signal processing for atmospheric radars, in Radar in Meteorology, edited by D. Atlas, chap. 20a, 199-229, American Meteorological Society, Boston, 1990.
2. Carter, D., Gage, K. S., Ecklund, W. L., Angevine, W. M., Johnston P. E., Riddle, A. C., Wilson, J., and Williams, C. R., Developments in

- UHF lower tropospheric wind profiling at NOAA's Aeronomy Laboratory, Radio Sci., 30, 977-1001, 1995.
3. Wilfong, T. L., Merritt, D. A., Weber, B. L., and Wuertz, D. B., Multiple signal detection and moment estimation in radar wind profiler spectral data, submitted to J. Atmos. Oceanic Technol., 1999b.
4. Schmidt, G., R'uster, R., and Czechowsky, P., Complementary code and digital filtering for detection of weak VHF radar signals from the Mesosphere, IEEE Trans. Geosci. Electron., GE-17, 154-161, 1979.
5. Sulzer, M. and Woodman, R., Quasi-complementary codes: A new technique for MST radar sounding, Radio Sci., 19, 337-344, 1984.
6. Spano, E. and Ghebrehhan, O., Pulse coding techniques for ST/MST radar systems: A general approach based on a matrix formulation, IEEE Trans. Geosci. Remote Sensing, 34, 304-316, 1996.
7. Tsuda, T., Middle Atmosphere Program - Handbook for MAP, vol. 30, chap. Data Acquisition and Processing, pp. 151-183, ICSU Scientific Committee on Solar-Terrestrial Physics (SCOSTEP), ISAR 24-28 November 1988, Kyoto, 1989.
8. May, P. T. and Strauch, R. G., Reducing the effect of ground clutter on wind profiler velocity measurements, J. Atmos. Oceanic Technol., 15, 579-586, 1998.
9. Gossard, E. E., A fresh look at the radar reflectivity of clouds, Radio Sci., 14, 1089-1097, 1979.
10. Gossard, E. E. and Strauch, R. G., The refractive index spectra within clouds from forward-scatter radar observations, J. Appl. Meteor., 20, 170-183, 1981. Gossard, E. E. and Strauch, R. G., Radar Observations of Clear Air and Clouds, Elsevier, 1983.
11. Ralph, F. M., Neiman, P. L., and Ruffieux, D., Precipitation identification from radar wind profiler spectral moment data: Vertical velocity histograms, velocity variance, and signal power - vertical velocity correlation, J. Atmos. Oceanic Technol., 13, 545-559, 1996.
12. Gage, K. S., Williams, C. R., Ecklund, W. L., and Johnston, P. E., Use of two profilers during MCTEX for unambiguous identification of Bragg scattering and Rayleigh scattering, J. Atmos. Sci., 56, 3679-3691, 1999. Ghebrehhan, O. and Crochet, M., On full decoding of truncated ranges for ST/MST radar applications, IEEE Trans. Geosci. Electron., 30, 38-45, 1992.
13. Daubechies, I., Ten Lectures on Wavelets, SIAM, Philadelphia, 1992.
14. Vetterli, M. and Kova'cevi'c, J., Wavelets and Subband Coding, Prentice Hall PTR, New Jersey, 1995.
15. Louis, A. K., Maaß, P., and Rieder, A., Wavelets, Teubner, Stuttgart, 1998. Stark, H.-G., Continuous wavelet transform and continuous multiscale analysis, Math. Anal. and Appl., 169, 179-196, 1992.
16. Meyer, Y., Wavelets: Algorithms and Applications, SIAM, Philadelphia, 1993. Monna, W. A. and Chadwick, R. B., Remote-sensing of upper-air winds for weather forecasting: Wind-profiler radar, Bull. WMO, 47, 124-132, 1998.
17. Holschneider, M., Wavelets: An Analysis Tool, Clarendon Press, Oxford, 1995.
18. Burrus, C. S., Gopinath, R. A., and Guo, H., Introduction to Wavelets and Wavelet Transforms, Prentice Hall, 1998.
19. Teschke, G., Komplexwertige Wavelets und Phaseninformation, Anwendungen in der Signalverarbeitung, Diplomarbeit, Institut für Mathematik, Universität Potsdam, 1998.
20. Dahlke, S. and Maaß, P., The affine uncertainty principle in one and two dimensions, Comp. Math. Appl., 30, 293-305, 1995.
21. Kaiser, G., A Friendly Guide to Wavelets, Birkh'auser, Basel, 1994.
22. Donoho, D. L. and Johnstone, I. M., Minimax estimation via wavelet shrinkage, Tech. Rep. 402, Department of Statistics, Stanford University, 1992.
23. Donoho, D. L., Johnstone, I. M., Kerkyacharian, G., and Picard, D., Density estimation by wavelet thresholding, Preprint, Dept. of Statistics, Stanford University, 1993.
24. Johnstone, I. M. and Silverman, B. W., Wavelet threshold estimators for data with correlated noise, Tech. Rep. Dept. of Statistics, Stanford University, 1995.
25. Dahlhaus, R., Neumann, M. H., and v. Sachs, R., Nonlinear wavelet estimation of time-varying autoregressive processes, preprint, 1998.