Fractional Fourier Domain MRI Reconstruction using Compressive Sensing Under Different Random Sampling Scheme

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Abstract— ***In clinical Magnetic Resonance Imaging (MRI), any reduction in scan time offers an improvement in patient comfort problem.*** **Compressive sensing introduces a new technique to image reconstruction from less amount of data. It will reduce imaging time in MRI. Compressive sensing exploit the sparsity of the signal. In this paper fractional Fourier is used as sparsifying transform and signal sampled using different random sampling method. Such as gaussian, bernoullie,and poisson distribution. Then MRI** ***accurately reconstructed from very highly under sampled data*** **using Maximum likelihood estimation.**

Index Terms— **Compressive sensing, Fractional Fourier transform, maximum likelihood estimation**

I. INTRODUCTION

Researches are going on to increase speed of data acquisition and reduce resource consumption due to measurements. Whatever the field of application, most of the acquisition systems built during the last 50 years have been designed under the guiding rules of the Nyquist-Shannon sampling theorem. The sampling rate must be at least twice the maximum frequency present in the signal (the so-called Nyquist rate) [1], [2]. Main disadvantages of conventional approach are exponentially increasing amount of data, data acquisition time is high ,energy consumption.

MRI is a noninvasive imaging modality to visualize internal organs[3]. The data-intensive nature MRI applications inherently prescribe a lengthening of scan duration which can decrease patient comfort, increase the risk of physiological artifacts, and reduce clinical throughput. As many MR images are piecewise smooth and thus naturally exhibit sparsity in the fractional Fourier domain, it is now accepted that accurate reconstruction of the constituent image structures can be achieved using a small subset of their fractional Fourier measurements.

CS is suitable for MRI, because MRI measurments are in Fourier or fractional Fourier domain. Compressive sampling will reduces imaging time in MRI by sampling much fewer measurements than Nyquistic rate . This paper considers the basic problem of recovering an orginal image, f in C^N from a fewer set of measurements. Previous article studied signals which have relatively less nonzero terms or whose coefficients in some fixed basis have relatively few nonzero entries[4]. This paper discussed some surprising phenomena, and has relatively very less nonzero coefficient than preveous. This work presents signal sparsity, sampling, signal coding and reconstruction. Actually compressive sensing is applied to the analog form of the signal, here orginal image is used for analysis and then generate sparse signal. For satisfying sparsity, fractional Fourier transform is applied as a sparsifying transform. Then signal is sampled using different random sampling method and then generate the orginal image by using maximum likelihood estimation. The formulation of CS theory emphasizes on maximizing image sparsity on known sparse transform domain and minimizing fidelity. Gaussian, Bernoulli or Poisson under sampling scheme is used as the random projection matrix for MRI problem.

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II. PRILIMINARIES

A. *Compressive sensing*

Compressive sensing suggests the possibility of new data acquisition protocols that translate analog information into digital form with fewer sensors than what was considered necessary. This new sampling theory may come to underlie procedures for sampling and compressing data simultaneously, in order to solve under-determined problem [8], [9]. The basics in this approach are that the signal to be sampled is sparse in a convenient basis. In this paper fractional Fourier transform serves as the sparsity basis. Mathematically, any signal f€R^n (such as the *n*-pixel image) can express as a linear combination of an orthonormal basis ψ = [ψ_1, ψ_2.........ψ_n] and coefficient as follows:

$$\mathbf{x} = \sum_{i=1}^{n} \psi_i \ \alpha \tag{1}$$

where ψ is sparsity promoting transform

CS we donot aquire x directly but rather aquire M<N measurements. MR imaging for instance, one is typically able to collect far fewer measurements about an image of interest than the number of unknown pixels. This is “underdetermined” case where we have many fewer measurements than unknown values. Solution for the underdetermined system of equations appears hopeless. For instance, suppose signal is sparse and it can be written either exactly or accurately as a superposition of a small number of vectors in some fixed basis. Then this concept radically changes the problem making the search for solutions feasible. In fact, accurate recovery is possible

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by solving a convex optimization problem.

The second premise of CS concerns the mutual coherence between the measurement and the sparsity basis[10]. Assume that the measurement matrix Φ and the sparsifying matrix ψ are orthonormal bases in C^N .

$$y=\Phi x \qquad (2)$$

B. *Fractional Fourier transform*

Fractional Fourier transform(FrFT) is a generalization of the Fourier transform. It transform a signal (either in the time domain or frequency domain) into the domain between time and frequency. The FrFT interpreted as a rotation by an angle $\alpha=a\pi/2$ in the time–frequency plane. An FrFT with $\alpha=\pi/2$ corresponds to the classical Fourier transform, and an FrFT with $\alpha=0$ corresponds to identity operator [11], [12].

$$\mathcal{F}_\alpha f(u) = \int K_\alpha(u,x) f(x)\, dx \qquad (3)$$

$$K_\alpha(u,x) = \begin{cases} \sqrt{1-\cot(\alpha)}\exp\left(i\pi\left(\cot(\alpha)\left(x^2+u^2\right)-2\csc(\alpha)ux\right)\right) & \text{if } \alpha \text{ is not multiple of } \pi \\ \delta(u-x) & \text{if } \alpha \text{ is multiple of } 2\pi \\ \delta(u+x) & \text{if } \alpha+\pi \text{ is multiple of } 2\pi \end{cases}$$

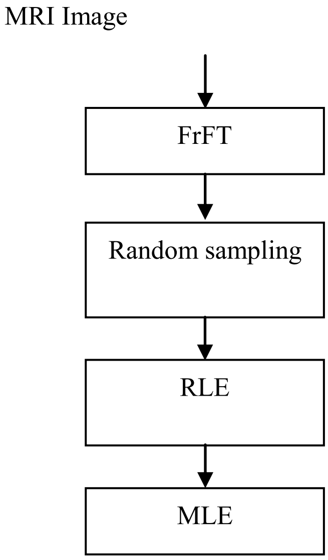
III. Proposed Model using Compressive Sensing

A. Data Acquisition

The original images were obtained on a 3T scanner with a dimension of 216x216. These images considered as the ground truth for this study. This image is first partitioned into non-overlapped 8X8 subimage.

B.*Sparsifying Transform*

Image is converted in to sparsifying domain using fractional Fourier transform. In this step, a 2D-FrFT is applied to each block to convert the gray levels of pixels in the spatial domain into coefficients in the frequency domain. By using FRFT a large amount of information is pack into smallest number of transform coefficients, hence small amount of compression is achieved at this step.The heart of the routine consists of three steps. Multiplications of f with a chirp function, this result is convolved with a chirp function and again multiplies with a chirp function.



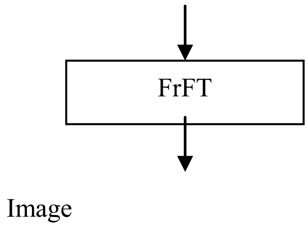


Fig 1 :Schematic diagram of Proposed method

A. Random Sampling

To satisfy incoherent under-sampling in CS, sampled using a random Bernoulli, Gaussian or Poisson probability function to keep only fewer amount of data.

B. Run legth encoding

Compressed datas are coded using run length encoding. Run length coding can be applied on a row-by-row basis. RLE is most suitable coding method, because most of the pixel values are same in MRI.

C. Reconstruction

Reconstruction is based on estimation theory. Estimation is a systematic way of searching for the parameter values of our chosen distribution. That maximizes the probability of observing the data [15].

Maximum likelihood esimator,

$$\hat{\theta} = \arg\max_{\theta} \sum_{i=1}^{n} \log\ p(x_i/\theta) \qquad (4)$$

The goal of MLE is to find values of the parameters, say β, which maximize the (log) likelihood function. To do this, we could start with a guess of β and let's call this $\beta 0$. We could then adjust this guess based on the value of the (log) likelihood that it gives us.

Thus, our new guess would be

$$\hat{\beta}_K = \hat{\beta}_{K-1} + \hat{\lambda}_{K-1}\,\hat{\Delta}_{K-1} \qquad (5)$$

$\hat{\Delta}_{K-1}$-Direction to take step

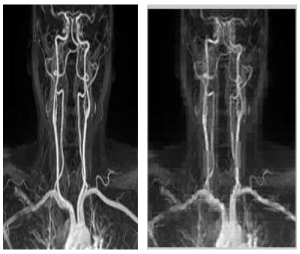
$\hat{\lambda}_{K-1}$-step size

Move the vector β to the point at which the likelihood is highest. Take account of the slope of the likelihood function at each guess. Intuitively, the way that we do this is by incorporating information from the gradient. As we get to the top, the gradient becomes closer to zero.Then we stop, that will be the estimate. It is proved that estimate will be mean of the samples.

Reconstruction of orginal image from estimated data can be done using inverse-FrFT with respect to angle θ . which is same as FrFT at angle $-\theta$.

For analyzing resulted image, we used Peak SNR (PSNR), MSE and Compression ratio measurements. The SSIM is a method for measuring the similarity between two images.

IV. THE IMPLIMENTATION AND RESULTS



(a) (b)

(c) (d)

Fig 2 (a) **Image used in this paper for experiments**, (b)Reconstructed image using bernoulli random sampling (c)gaussian sampling (d)poisson sampling

TABLE I. Performance analysis

sampling	PSNR	MSE	Compression ratio
Bernoulli	70.76	0.003	1.7
Poisson	70.5	0.0058	2
Gaussian	71.4	0.005	2.2

V. CONCLUSION

Fractional Fourier transform and Maximum likelihood estimator are used to reconstruct high quality images from few MR compressive sensing measurements under different sampling pattern. CS changes the rules of the data acquisition game. We can reduce acquisition time in MRI using CS principle. It leads to next generation data acquisition . Analysis done on PSNR and compression ratio. Gaussian sampling have better perfomance under this class of images. Future projects can use adaptive sparse basis and Under-sampling pattern such as Poisson disc sampling can be used as undersampling matrix.

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