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Effect of Hall Current on Thermal Convection of Micro polar Fluid Layer in The Presence of Horizontal Magnetic Field Saturating a Porous Medium

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Abstract-This paper deals with the theoretical investigation of Hall current effect on micro polar fluid layer heated from below subjected to horizontal magnetic field in a porous medium. A dispersion relation is obtained for a flat fluid layer contained between two free boundaries using a linear stability analysis theory and normal mode analysis method. In case of stationary convection, the effect of various parameters like medium permeability, coupling parameter, micropolar coefficient, micro polar heat conduction parameter, magnetic field and Hall current parameter has been analyzed and results are depicted graphically. The sufficient conditions for non-existence of over stability are also obtained.

Keywords:- Thermal convection, Micropolar fluid, Horizontal magnetic field, Hall current, Porous Medium

I. INTRODUCTION

Chandrasekhar[11] has studied a thermal convection in a horizontal thin layer of Newtonian fluid heated from below, under varying assumptions of hydrodynamics. **Ahmadi[5]** and **Pérez-Garcia** *et. al*[**3**] have studied the effects of the microstructures on the thermal convection, they have found that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stability holds. **Pérez-Garcia** and **Rubi[2]** have found that when the coupling between thermal and micropolar effects is present, the principle of exchange of stability may not be fulfilled and hence that oscillatory modes are present in micropolar fluid. **Sharma** and **Kumar[8]** have studied the effect of magnetic field on the micropolar fluids heated from below. In the presence of strong electric field, the electric conductivity is affected by the magnetic field. Thus, the parallel to the electric field is reduced and hence, the current is reduced in direction normal to both electric and magnetic fields. This phenomenon is known as the **Hall effect**. In an ionized gas, when the strength of magnetic field is large, one can not neglect the effect of Hall current. These Hall currents are particularly important as produce considerable changes in the flow pattern, when the magnetic field is considerable large. **A.S. Gupta[1]** has studied the Hall current effects in the steady hydromagnetic flow of an electrically conducting fluid past an infinite porous flat plate. **Raghavachar** and **Gothandaraman[7]** have studied hydromagnetic convection in a rotating fluid layer in the presence of Hall current. **Aboeldahab** and **Elbarby[4]** examined Hall current effects on MHD free convection flow past a semi-infinite vertical plate with mass transfer. **Acharya** *et. al.*[**6**] have studied Hall current effects with simultaneous thermal and mass diffusion on unsteady hydromagnetic flow near an accelerated vertical plate. **Takhar[13]** has studied unsteady flow free convective flow over an infinite vertical porous plate due to combined effects of thermal and mass diffusion, magnetic fluid and Hall current. The effect of Hall current on thermal instability has also been studied by **Sharma** and **Gupta[9]**, **Sharma** *et. al*[**10**], **Sunil** *et. al* [**12**], **Gupta** and **Aggarwal[14, 15]**.

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II. MATHEMATICAL FORMULATION

We consider an infinite, horizontal, electrically non-conducting, incompressible micropolar fluid layer of thickness *d*. A cartesian co-ordinate system (*x*, *y*, *z*) is choosen in such a way that the origin is at the lower boundary and the *z*-axis is vertically upward. This layer is heated from below such that the lower boundary is held at constant temperature $T = T_0$ and the upper boundary is held at fixed temperature $T = T_1$ so that $T_0 > T_1$, therefore a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained. The physical structure of the problem is one of infinite extent in *x* and *y* directions bounded by the planes $z = 0$ and $z = d$.

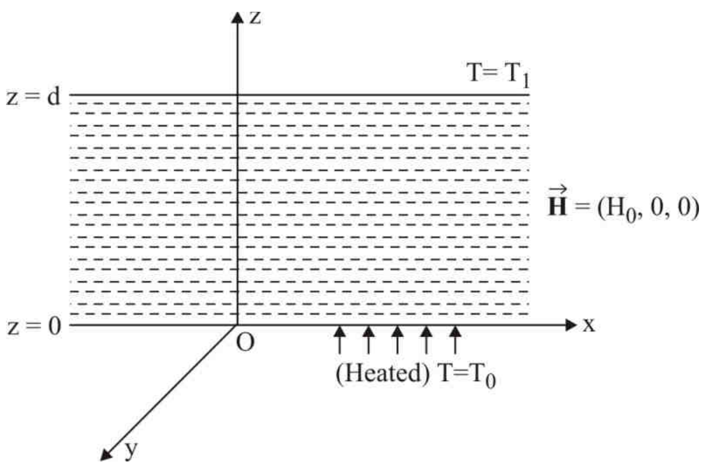

Fig. 1

This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of poricity $\in$ and the medium permeability κ. The whole system is acted upon by a gravity field $\vec{\mathbf{g}} = (0, 0, -g)$ and a strong but uniform magnetic field $\vec{\mathbf{H}} = (H_0, 0, 0)$, H_0 is a constant, is applied along *x*-direction. The magnetic Raynold number is assumed to be very small so that the induced magnetic field can be neglected in comparison to the applied field. We also assumed that both the boundaries are free and no external couples and heat sources are present. For an isotropic medium the surface porosity is $\in$ so that $1-\in$ is the fraction that is occupied by solid. Within Boussinesq approximation, the equations governing the motion of a micropolar fluid saturating porous medium are as follows:The equation of continuity for an incompressible fluid is

$$\nabla.\vec{\mathbf{q}} = 0 \quad ...(1)$$

The equation of momentum following Darcy law is given by

$$\frac{\rho_o}{\in}\left[\frac{\partial \vec{\mathbf{q}}}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}.\nabla)\vec{\mathbf{q}}\right] = -\nabla p - \rho q \hat{e}_z + (\mu+\zeta)\nabla^2 \vec{\mathbf{q}}$$

$$-\frac{\mu}{\kappa}\vec{\mathbf{q}} + \zeta\nabla\times\vec{\mathbf{N}} + \frac{\mu_e}{4\pi}(\nabla\times\vec{\mathbf{H}})\times\vec{\mathbf{H}} \quad ..(2)$$

The equation of internal momentum is given by

$$\rho_o\, j\left[\frac{\partial \vec{\mathbf{N}}}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}.\nabla)\vec{\mathbf{N}}\right] = (\alpha'+\beta')\nabla(\nabla.\vec{\mathbf{N}}) + \gamma'\nabla^2\vec{\mathbf{N}}$$

$$+\zeta\left(\frac{1}{\in}\nabla\times\vec{\mathbf{q}} - 2\vec{\mathbf{N}}\right) \quad ...(3)$$

The equation of energy is given by

$$[\rho_o C_v \in + \rho_s C_s(1-\in)]\frac{\partial T}{\partial t} + \rho_o C_v(\vec{\mathbf{q}}.\nabla)T$$

$$= \chi_T \nabla^2 T + \delta(\nabla\times\vec{\mathbf{N}}).\nabla T \quad ...(4)$$

and the equation of state is given by

$$\rho = \rho_o[1-\alpha(T-T_0)] \quad ...(5)$$

Where $\vec{\mathbf{q}}, \vec{\mathbf{N}}, p, \rho, \rho_o\ \rho_s, \mu, \zeta, \kappa, \mu_e, j, \alpha', \beta', \gamma'$ $T, T_0, t, \chi_T, \delta, \alpha, C_v, C_s$ and $\hat{e}_z$ denote respectively filter velocity, microrotation, pressure, fluid density, reference density, density of solid matrix, viscosity, coupling viscosity, medium permeability, magnetic permeability, micro-inertia coefficient, micropolar viscosity coefficients, temperature, reference temperature, time, thermal conductivity, micropolar heat conduction coefficient, coefficient of thermal expansion, specific heat at constant volume, specific heat of solid matrix and unit vector along *z*-direction.

The Maxwell's equation with Hall current is given by

$$\in\frac{\partial H}{\partial t} = \nabla\times(\vec{\mathbf{q}}\times\vec{\mathbf{H}}) + \in\gamma_m\nabla^2\vec{\mathbf{H}} - \frac{\in}{4\pi e n_e}\nabla\times\left[(\nabla\times\vec{\mathbf{H}})\times\vec{\mathbf{H}}\right] ...(6)$$

and $\nabla.\vec{\mathbf{H}} = 0 \quad ...(7)$

Where γ_m, e and n_e denote respectively magnetic viscosity, charge on electron, and electron number density.

III. BASIC STATE OF THE PROBLEM

The basic state of the problem is taken as $\vec{\mathbf{q}} = \vec{\mathbf{q}}_b = (0,0,0)$, $\vec{\mathbf{N}} = \vec{\mathbf{N}}_b = (0,0,0)$, $p = p_b(z)$,

$\rho = \rho_b(z)$, $\vec{\mathbf{H}} = \vec{\mathbf{H}}_b = (H_0, 0, 0)$. Using above basic state, equations (1) to (7) become

$$\frac{dp_b}{dz} + \rho_b g = 0 \quad ...(8)$$

$$T = T_b = -\beta z + T_0 \quad ...(9)$$

and $\rho_b = \rho_o(1+\alpha\beta z) \quad ...(10)$

IV. PERTURBATION EQUATIONS

After perturbation, the new variables become

$$\vec{\mathbf{q}} = \vec{\mathbf{q}}_b + \vec{\mathbf{q}}',\ \vec{\mathbf{N}} = \vec{\mathbf{N}}_b + \vec{\mathbf{N}},\ p = p_b + p',\ \rho = \rho_b + \rho',$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}_b + \vec{\mathbf{h}},\ T = T_b + \theta \quad ...(11)$$

Where $\vec{\mathbf{q}}', \vec{\mathbf{N}}', p', \rho', \vec{\mathbf{h}}$ and θ are the perturbations in $\vec{\mathbf{q}}, \vec{\mathbf{N}}, p, \rho, \vec{\mathbf{H}}$ and T respectively.

Using (8)-(11), equation (1)-(7) yield

$$\nabla.\vec{\mathbf{q}}' = 0 \quad ...(12)$$

$$\frac{\rho_o}{\in}\left[\frac{\partial \vec{\mathbf{q}}'}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{q}}'\right] = -\nabla p' + (\mu+\zeta)\nabla^2\vec{\mathbf{q}}' - \frac{\mu}{K}\vec{\mathbf{q}}' - \rho' q\,\hat{e}_z + \zeta\nabla\times\vec{\mathbf{N}}'$$

$$+\frac{\mu_e}{4\pi}(\nabla\times\vec{\mathbf{h}})\times\vec{\mathbf{H}}_b + \frac{\mu_e}{4\pi}(\nabla\times\vec{\mathbf{h}})\times\vec{\mathbf{h}} \quad ...(13)$$

$$\rho_o\, j\left[\frac{\partial \vec{\mathbf{N}}}{\partial t} + \frac{1}{\in}(\vec{\mathbf{q}}.\nabla)\vec{\mathbf{N}}'\right] = (\alpha'+\beta')\nabla(\nabla.\vec{\mathbf{N}}') + \gamma'\nabla^2\vec{\mathbf{N}}') + \zeta\left(\frac{1}{\in}\nabla\times\vec{\mathbf{q}}' - 2\vec{\mathbf{N}}'\right)$$

..(14)

$$[\rho_o C_v \in + \rho_s C_s(1-\in)]\frac{\partial\theta}{\partial t} + \rho_o C_v(\vec{\mathbf{q}}'.\nabla)\theta + \rho_o C_v(\vec{\mathbf{q}}'.\nabla)T_b$$

$$= \chi_T\nabla^2\theta + \delta(\nabla\times\vec{\mathbf{N}}').\nabla\theta + \delta(\nabla\times\vec{\mathbf{N}}').\nabla T_b \quad ...(15)$$

$$\in\frac{\partial\vec{\mathbf{h}}}{\partial t} = \nabla\times(\vec{\mathbf{q}}'\times\vec{\mathbf{h}}) + \nabla\times(\vec{\mathbf{q}}'\times\vec{\mathbf{H}}_b) + \in\gamma_m\nabla^2\vec{\mathbf{h}}$$

$$-\frac{\in}{4\pi n_e e}\nabla\times\left[(\nabla\times\vec{\mathbf{h}})\times\vec{\mathbf{h}} + (\nabla\times\vec{\mathbf{h}})\times\vec{\mathbf{H}}_b\right] \quad ..(16)$$

$$\nabla.\vec{\mathbf{h}} = 0 \quad ...(17)$$

and $\rho = -\rho_o\alpha\theta \quad ...(18)$

In order to linearlize above equations, we ignore non-linear terms

$(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{q}}'$, $(\vec{\mathbf{q}}'.\nabla)\vec{\mathbf{N}}'$, $(\vec{\mathbf{q}}'.\nabla)\theta$, $(\nabla\times\vec{\mathbf{h}})\times\vec{\mathbf{h}}$, $(\nabla\times\vec{\mathbf{N}}').\nabla\theta$, $\nabla\times(\vec{\mathbf{q}}'\times\vec{\mathbf{h}})$. The linearlized equations are

$$\nabla.\vec{\mathbf{q}}' = 0 \qquad ...(19)$$

$$\frac{\rho_o}{\in}\frac{\partial \vec{\mathbf{q}}'}{\partial t} = -\nabla p' + (\mu+\zeta)\nabla^2\vec{\mathbf{q}}' - \frac{\mu}{\kappa}\vec{\mathbf{q}}' - \rho' g\,\hat{e}_z + \zeta\nabla\times\vec{\mathbf{N}}' + \frac{\mu_e H_o}{4\pi}(\nabla\times\vec{\mathbf{h}})\times\hat{e}_x \qquad ...(20)$$

$$\rho_o\, j\frac{\partial N}{\partial t} = (\alpha'+\beta'+\gamma')\nabla(\nabla.\vec{\mathbf{N}}') - \gamma'\nabla\times(\nabla\times\vec{\mathbf{N}}') + \zeta\left(\frac{1}{\in}\nabla\times\vec{\mathbf{q}}' - 2\vec{\mathbf{N}}'\right) \qquad ...(21)$$

$$\left[\rho_o C_v \in + \rho_s C_s(1-\in)\right]\frac{\partial\theta}{\partial t} + \rho_o C_v(\vec{\mathbf{q}}'.\nabla)T_b = \chi_T\nabla^2\theta + \delta(\nabla\times\vec{\mathbf{N}}').\nabla T_b \qquad ...(22)$$

$$\in\frac{\partial\vec{\mathbf{h}}}{\partial t} = H_o\nabla\times(\vec{\mathbf{q}}'\times\hat{e}_x) + \in\gamma_m\nabla^2\vec{\mathbf{h}} - \frac{\in H_o}{4\pi e n_e}\nabla\times\left[(\nabla\times\vec{\mathbf{h}})\times\hat{e}_x\right] \qquad ...(23)$$

$$\nabla.\vec{\mathbf{h}} = 0 \qquad ...(24)$$

$$\rho' = -\rho_o\alpha\theta \qquad ...(25)$$

Making the equations (19)-(25) non-dimensional by using following transformations and dropping the stars, we have

$$\left.\begin{array}{l} x = dx^*,\, y = dy^*,\, z = dz^*,\, \vec{\mathbf{q}}' = \dfrac{K_T}{d}\vec{\mathbf{q}}^*,\, \vec{\mathbf{N}} = \dfrac{K_T}{d^2}\vec{\mathbf{N}}^*,\, t = \dfrac{\rho_o d^2}{\mu}t^* \\ \theta = \beta d\theta^*,\, p' = \dfrac{\mu K_T}{d^2}p^*,\, K_T = \dfrac{\chi_T}{\rho_o C_v},\, \vec{\mathbf{h}} = H_o\vec{\mathbf{h}}^* \end{array}\right] \qquad ..(26)$$

where K_T is the thermal diffusivity and dropping the stars, we have

$$\frac{1}{\in}\frac{\partial\vec{\mathbf{q}}}{\partial t} = -\nabla p + R\theta\,\hat{e}_z + (1+K)\nabla^2\vec{\mathbf{q}} - \frac{1}{K_1}\vec{\mathbf{q}} + K\nabla\times\vec{\mathbf{N}} + Q(\nabla\times\vec{\mathbf{h}})\times\hat{e}_x \qquad ..(27)$$

$$\bar{\mathbf{j}}\frac{\partial\vec{\mathbf{N}}}{\partial t} = C'\nabla(\nabla.\vec{\mathbf{N}}) - C\nabla\times(\nabla\times\vec{\mathbf{N}}) + K\left(\frac{1}{\in}\nabla\times\vec{\mathbf{q}} - 2\vec{\mathbf{N}}\right) \qquad ..(28)$$

$$EP_r\frac{\partial\theta}{\partial t} = \nabla^2\theta + \omega - \bar{\boldsymbol{\delta}}\,\xi \qquad ..(29)$$

$$\in P_r\frac{\partial\vec{\mathbf{h}}}{\partial t} = \frac{\partial\vec{\mathbf{q}}}{\partial x} + \frac{\in P_r}{P_m}\nabla^2\vec{\mathbf{h}} - \in\beta_e^{1/2}\frac{\partial}{\partial x}(\nabla\times\vec{\mathbf{h}}) \qquad ..(30)$$

Where $R = \dfrac{\rho_o g\alpha\beta d^4}{\mu K_T}$ is the thermal Rayleigh number, $Q = \dfrac{\mu_e H_0^2 d^2}{4\pi\mu K_T}$ is the Chandrasekhar number. $C' = \dfrac{\alpha'+\beta'+\gamma'}{\mu d^2}$, $C = \dfrac{\gamma'}{\mu d^2}$, $K = \dfrac{\zeta}{\mu}$, $K_1 = \dfrac{\kappa}{d^2}$, $E = \in + \dfrac{\rho_s C_s(1-\in)}{\rho_o C_v}$, $P_r = \dfrac{\mu}{\rho_o K_T}$ is the parandtl number, $P_m = \dfrac{\mu}{\rho_o\gamma_m}$ is the magnetic prandtl number, $\xi = (\nabla\times N).\hat{e}_z$, $w = \mathbf{q}.\hat{e}_z$ and $\beta_e = \left(\dfrac{H_0}{4\pi K_T e n_e}\right)^2$ is the Hall parameter, and $\bar{\boldsymbol{\delta}} = \dfrac{\delta}{\rho_o C_v d^2}$ is the coupling parameter.

V. BOUNDARY CONDITIONS

Suppose that both the boundaries of this problem are free and perfectly heat conducting, thus the boundary conditions are

$$w = 0 = \frac{\partial^2 w}{\partial z^2},\ \theta = 0,\ \vec{\mathbf{N}} = 0,\ \xi = 0 \text{ at } z = 0 \text{ and } z = 1 \qquad ...(31)$$

VI. DISPERSION RELATIONS

Taking curl twice on both sides of (27), and taking z-component, we have

$$\frac{1}{\in}\frac{\partial}{\partial t}(\nabla^2 w) = R\nabla_1^2\theta + (1+K)\nabla^4 w - \frac{1}{K_1}\nabla^2 w + K\nabla^2\xi + Q\frac{\partial}{\partial x}\nabla^2 h_z \qquad ...(32)$$

Taking curl to equation (28) and taking *z*-component, we have

$$\bar{\mathbf{j}}\frac{\partial\xi}{\partial t} = C\nabla^2\xi - K\left(\frac{1}{\in}\nabla^2 w + 2\xi\right) \qquad ...(33)$$

Taking curl to equation (30) and taking *z*-component, we have

$$\in P_r\frac{\partial m_z}{\partial t} = \frac{\partial\zeta_z}{\partial x} + \frac{\in P_r}{P_m}\nabla^2 m_z + \beta_e^{1/2}\frac{\partial}{\partial x}\nabla^2 h_z \qquad ...(34)$$

Taking *z*-component of equation (30), we have

$$\in P_r\frac{\partial h_z}{\partial t} = \frac{\partial w}{\partial x} + \frac{\in P_r}{P_m}\nabla^2 h_z - \in\beta_e^{1/2}\frac{\partial m_z}{\partial x} \qquad ...(35)$$

Taking curl on both sides of (27), we have

$$\frac{1}{\in}\frac{\partial}{\partial t}(\nabla\times\mathbf{q}) = R\left[\frac{\partial\theta}{\partial y}\hat{e}_x - \frac{\partial\theta}{\partial x}\hat{e}_y\right] + (1+K)\nabla^2(\nabla\times\mathbf{q}) - \frac{1}{K_1}(\nabla\times\mathbf{q}) + K\nabla\times(\nabla\times\vec{\mathbf{N}}) + Q\frac{\partial}{\partial x}(\nabla\times\vec{\mathbf{h}}) \qquad ...(36)$$

Taking curl twice to equation (28) and taking *z*-component, we have

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$$\left[\bar{\mathbf{j}}\frac{\partial}{\partial t}-C\nabla^2+2K\right](\nabla\times\nabla\times N).\hat{e}_z=-K\nabla^2\zeta_z \qquad ...(37)$$

Eliminating $(\nabla\times\nabla\times\vec{\mathbf{N}}).\hat{e}_z$ between (36) and (37), we have

$$\left\{\left[\frac{1}{\in}\frac{\partial}{\partial t}-(1+K)\nabla^2+\frac{1}{K_1}\right]\left[\bar{\mathbf{j}}\frac{\partial}{\partial t}-C\nabla^2+2K\right]+K^2\nabla^2\right\}\zeta_z$$

$$=Q\left[\bar{\mathbf{j}}\frac{\partial}{\partial t}-C\nabla^2+2K\right]\frac{\partial m_z}{\partial x} \qquad ...(38)$$

and $$EP_r\frac{\partial\theta}{\partial t}=\nabla^2\theta+w-\bar{\boldsymbol{\delta}}\xi \qquad ...(39)$$

Where

$\nabla_1^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$, $h_z=\vec{\mathbf{h}}.\hat{e}_z$, $\zeta_z=(\nabla\times\vec{\mathbf{q}}).\hat{e}_z$, $m_z=(\nabla\times\vec{\mathbf{h}}).\hat{e}_z$, $\xi=(\nabla\times\vec{\mathbf{N}}).\hat{e}_z$ and $\hat{e}_x$, $\hat{e}_y$ and $\hat{e}_z$ are the unit vectors along x, y and z axes respectively.

VII. NORMAL MODE ANALYSIS

The normal mode analysis can be defined as

$[w,\ \zeta_z,\ \xi,\ \theta,\ h_z,\ m_z]$

$=[W(z),\ X(z),\ G(z),\ \Theta(z),\ B(z),\ M(z)]\exp.(ik_x x+ik_y y+\sigma t)$

Using this normal mode analysis equations (32)-(35) and (38)-(39) become

$$\frac{\sigma}{\in}(D^2-a^2)W=-Ra^2\Theta+(1+K)(D^2-a^2)^2W$$

$$-\frac{1}{K_1}(D^2-a^2)W+K(D^2-a^2)G+Qik_x(D^2-a^2)B \qquad ...(40)$$

$$[\bar{j}\sigma-C(D^2-a^2)+2K]G=-\frac{K}{\in}(D^2-a^2)W \qquad ...(41)$$

$$\left[EP_r\sigma-(D^2-a^2)\right]\left[\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right]\Theta$$

$$=\left\{\left[\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right]+\frac{K\bar{\boldsymbol{\delta}}}{\in}(D^2-a^2)\right\}W \qquad ...(42)$$

$$\left\{\left[\left\{\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right\}\left\{\frac{\sigma}{\in}-(1+K)(D^2-a^2)+\frac{1}{K_1}\right\}\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}\right.\right.$$

$$\left.+K^2\left\{\in P_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right\}(D^2-a^2)+k_x^2Q\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}\right]$$

$$\times\left[\in P_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right]-\in^2k_x^2\beta_e\left[\left\{\frac{\sigma}{\in}-(1+K)(D^2-a^2)+\frac{1}{K_1}\right\}\right.$$

$$\left.\left.\times\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}(D^2-a^2)+K^2(D^2-a^2)^2\right]\right\}B$$

$$=ik_x\left[\left\{\in P_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right\}\left\{\frac{\sigma}{\in}-(1+K)(D^2-a^2)+\frac{1}{K_1}\right\}\right.$$

$$\times\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}+K^2\left\{\in P_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right\}(D^2-a^2)$$

$$\left.+k_x^2Q\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}\right]W \qquad ...(43)$$

Eliminating Θ, G, B between (40), (41), (42) and (43), we have

$$\left[\left\{\in P_r\sigma-\in\frac{P_r}{P_m}\left(D^2-a^2\right)\right\}^2\left\{\frac{\sigma}{\in}-(1+K)(D^2-a^2)+\frac{1}{K_1}\right\}\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}\right.$$

$$+K^2\left\{\in P_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right\}^2(D^2-a^2)$$

$$\times\left\{\in P_r\sigma-\in\frac{P_r}{P_m}(D^2-a^2)\right\}$$

$$-\in^2k_x^2\beta_e\left[\left\{\frac{\sigma}{\in}-(1+K)(D^2-a^2)+\frac{1}{K_1}\right\}\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}\right.$$

$$\left.\left.\times(D^2-a^2)+K^2(D^2-a^2)\right]\right]\left[\left\{\frac{\sigma}{\in}(D^2-a^2)-(1+K)(D^2-a^2)^2+\frac{1}{K_1}(D^2-a^2)\right\}\right.$$

$$\times\left\{EP_r\sigma-(D^2-a^2)\right\}\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}$$

$$+Ra^2\left\{\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}+\frac{K\bar{\boldsymbol{\delta}}}{\in}(D^2-a^2)\right\}$$

$$\left.+\frac{K^2}{\in}(D^2-a^2)^2\left\{EP_r\sigma-(D^2-a^2)^2\right\}\right]W$$

$$=-k_x^2Q(D^2-a^2)\left\{EP_r\sigma-(D^2-a^2)\right\}\left\{\bar{j}\sigma-C(D^2-a^2)+2K\right\}$$

$$\times\left[\left\{\in P_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right\}\left\{\frac{\sigma}{\in}-(1+K)(D^2-a^2)+\frac{1}{K_1}\right\}\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}\right.$$

$$\left.+K^2(D^2-a^2)\left\{\in P_r\sigma-\frac{\in P_r}{P_m}(D^2-a^2)\right\}+k_x^2Q\left\{\bar{\mathbf{j}}\sigma-C(D^2-a^2)+2K\right\}\right]W$$

$$...(44)$$

The boundary conditions (31) reduce to

$W=0=D^2W=\Theta=M=DX=DM=B$ at $z=0$ and $z=1$

. ..(45)

Where $D\equiv\frac{\partial}{\partial z}$

Using boundary conditions (45) we obtain

$D^{(2n)}W=0$ at $z=0$ and $z=1$, where n is a positive integer. Thus, the proper solution W satisfying (45) can be taken as $W=W_o\sin\pi z$, Where W_o is a constant.

Substituting for W in equation (44), we have

$$\left[\left\{\in P_r\sigma+\frac{\in P_r}{P_m}b\right\}^2\left\{\frac{\sigma}{\in}+(1+K)b+\frac{1}{K_1}\right\}\left\{\bar{\mathbf{j}}\sigma+Cb+2K\right\}\right.$$

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$$-K^2b\left\{\in P_r\sigma+\frac{\in P_r}{P_m}b\right\}^2+k_x^2Q\{\bar{\mathbf{j}}\sigma+Cb+2K\}\left\{\in P_r\sigma+\frac{\in P_r}{P_m}b\right\}$$

$$-\in^2k_x^2\beta_e\Bigg[\Bigg[\left\{\frac{\sigma}{\in}+(1+K)b+\frac{1}{K_1}\right\}\{\bar{\mathbf{j}}\sigma+Cb+2K\}(-b)+K^2b\Bigg]\Bigg]$$

$$\times\Bigg[\Bigg[\left\{-\frac{\sigma}{\in}b-(1+K)b^2-\frac{b}{K_1}\right\}\{EP_r\sigma+b\}\{\bar{\mathbf{j}}\sigma+Cb+2K\}$$

$$+Ra^2\left\{\bar{\mathbf{j}}\sigma+Cb+2K-\frac{K\bar{\boldsymbol{\delta}}b}{\in}\right\}+\frac{K^2b^2}{\in}(EP_r\sigma+b)\Bigg]$$

$$=k_x^2Qb(EP_r\sigma+b)(\bar{\mathbf{j}}\sigma+Cb+2k)\Bigg[\Bigg[\left(\in P_r\sigma+\frac{\in P_r b}{P_m}\right)\left\{\frac{\sigma}{\in}+(1+K)b+\frac{1}{k_1}\right\}$$

$$\times(\bar{j}\sigma+Cb+2K)-K^2b\left(\in P_r\sigma+\frac{\in P_r}{P_m}b\right)+k_x^2Q(\bar{\mathbf{j}}\sigma+Cb+2K)\Bigg] \quad ..(46)$$

VIII. STATIONARY CONVECTION

Put $\sigma=0$ in (46), we have

$$R=\left\{\frac{\begin{array}{l}\Bigg[b^2\left\{(1+K)b+\frac{1}{K_1}\right\}(Cb+2K)-\frac{K^2b^3}{\in}\Bigg]\Bigg[\left\{(1+K)b+\frac{1}{K_1}\right\}(Cb+2K)-K^2b\\ +\frac{P_mk_x^2Q}{\in P_r b}(Cb+2K)+\left(\frac{P_m}{P_r}\right)^2\frac{k_x^2\beta_e}{b}\left\{\left((1+K)b+\frac{1}{K_1}\right)(Cb+2K)-K^2b\right\}\Bigg]\\ +\frac{P_mk_x^2Q}{\in P_r}\Bigg[b\left\{(1+K)b+\frac{1}{K_1}\right\}(Cb+2K)-K^2b^2+\frac{P_mk_x^2Q}{\in P_r}(Cb+2K)\Bigg]\end{array}}{\begin{array}{l}a^2\left(Cb+2K-\frac{K\bar{\delta}b}{\in}\right)\Bigg[\left\{(1+K)b+\frac{1}{K_1}\right\}(Cb+2K)-K^2b+\frac{k_x^2QP_m}{\in P_r b}(Cb+2K)\\ \qquad+\left(\frac{P_m}{P_r}\right)^2\frac{k_x^2\beta_e}{b}\left\{\left((1+K)b+\frac{1}{K_1}\right)(Cb+2K)-K^2b\right\}\Bigg]\end{array}}\right\} \quad ...(47)$$

In the absence of Hall current $(\beta_e=0)$ (and $H_0=0$) and in non-porous medium and in the absence of coupling parameter $(\bar{\boldsymbol{\delta}}=0)$, the equation (47) reduces to the equation proposed by **Goodarz Ahmadi [5].**

In the absence of Hall current and in non-porous medium, the equation (47) reduces to the equation proposed by **L.E. Payne** and **B. Straughan [16]** and **Y. Qin** and **P.N. Kaloni [17]**.

Equation (47) can be written as

$$R=\left\{\frac{\begin{array}{l}\Bigg[b^2\left\{(1+K)b+\frac{1}{K_1}\right\}(b+2A)-\frac{KAb^3}{\in}\Bigg]\Bigg[b\left\{(1+K)b+\frac{1}{K_1}\right\}(b+2A)-KAb^2\\ +\frac{P_mk_x^2Q}{\in P_r}(b+2A)+\frac{P_m^2k_x^2\beta_e}{P_r^2}\left\{\left((1+K)b+\frac{1}{K_1}\right)(b+2A)-KAb\right\}\Bigg]\\ +\frac{P_mk_x^2Qb}{\in P_r}\Bigg[b\left\{(1+K)b+\frac{1}{K_1}\right\}(b+2A)-KAb^2+\frac{P_mk_x^2Q}{\in P_r}(b+2A)\Bigg]\end{array}}{\begin{array}{l}a^2\left(b+2A-\frac{A\bar{\boldsymbol{\delta}}b}{\in}\right)\Bigg[b\left\{(1+K)b+\frac{1}{K_1}\right\}(b+2A)-KAb^2+\frac{k_x^2QP_m}{\in P_r}(b+2A)\\ +\frac{P_m^2k_x^2\beta_e}{P_r^2}\left\{\left(1+K)b+\frac{1}{K_1}\right)(b+2A)-KAb\right\}\Bigg]\end{array}}\right\} \quad ...(48)$$

Where $A=\dfrac{K}{C}$ is the micropolar coefficient.

In order to investigate the effects of medium permeability K_1, coupling parameter K, Micropolar coefficient A, Micropolar heat conduction parameter $\bar{\boldsymbol{\delta}}$, Magnetic field Q, and Hall current β_e, we examine the behaviour of $\dfrac{dR}{dK_1}, \dfrac{dR}{dK}, \dfrac{dR}{dA}, \dfrac{dR}{d\bar{\delta}}, \dfrac{dR}{dQ}$ and $\dfrac{dR}{d\beta_e}$.

From (48), we have

$$\frac{dR}{dK_1}=\frac{\begin{array}{l}-\frac{b^2(b+2A)}{K_1^2}\Bigg[\Bigg[\left(b+\frac{P_m^2k_x^2\beta_e}{P_r^2}\right)\left\{\left((1+K)b+\frac{1}{K_1}\right)(b+2A)-KAb\right\}+\frac{k_x^2QP_m(b+2A)}{\in P_r}\Bigg]^2\\ -\frac{P_m^4k_x^6Q^2\beta_e(b+2A)}{\in^2P_r^4b}\Bigg]\end{array}}{a^2\left(b+2A-\frac{A\bar{\delta}b}{\in}\right)\left[\begin{array}{l}\left(b+\frac{P_m^2k_x^2\beta_e}{P_r^2}\right)\left\{\left((1+K)b+\frac{1}{K_1}\right)(b+2A)-KAb\right\}\\ +\frac{k_x^2QP_m(b+2A)}{\in P_r}\end{array}\right]^2} \quad ...(49)$$

From (49) it is clear that $\dfrac{dR}{dK_1}<0$ when $\bar{\boldsymbol{\delta}}<\dfrac{\in}{A}$, and

$$\sqrt{\beta_e}<\frac{\sqrt{b^2+2Ab}}{P_m k_x}P_r.$$

Thus, the medium permeability has destabilizing effect when $\bar{\boldsymbol{\delta}}<\dfrac{\in}{A}$, and $\beta_e<\dfrac{P_r^2(b^2+2Ab)}{P_m^2k_x^2}$.

In the absence of Hall current $(\beta_e=0)$, it is clear that $\dfrac{dR}{dK_1}<0$ when $\bar{\boldsymbol{\delta}}<\dfrac{\in}{A}$, thus in this case medium

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permeability has destabilizing effect when $\bar{\boldsymbol{\delta}} < \dfrac{\in}{A}$.

From (48), we have

$$\frac{dR}{dK} = \frac{b^3\left[b+2A-\dfrac{A}{\in}\right]}{a^2\left(b+2A-\dfrac{A\bar{\boldsymbol{\delta}}b}{\in}\right)} - \frac{\dfrac{P_m^4 k_x^6 Q^2 \beta_e b^3 (b+A)(b+2A)}{\in^2 P_r^4}}{a^2\left(b+2A-\dfrac{A\bar{\boldsymbol{\delta}}b}{\in}\right)\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\times\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]^2}$$

It is clear that $\dfrac{dR}{dK} < 0$, when $\bar{\boldsymbol{\delta}} < \dfrac{\in}{A}$ and $0 < \in < \dfrac{1}{2}$ and $\dfrac{dR}{dK} > 0$ when $\dfrac{1}{2} < \in < 1, \bar{\boldsymbol{\delta}} < \dfrac{b}{2}$

Thus, the coupling parameter K has destabilizing effect when $\bar{\boldsymbol{\delta}} < \dfrac{\in}{A}$ and $0 < \in < \dfrac{1}{2}$ and it will have stabilizing effect when $\dfrac{1}{2} < \in < 1$ and $\bar{\boldsymbol{\delta}} < \dfrac{b}{2}$.

From (48), we have

$$\frac{dR}{dA} = \frac{\dfrac{\bar{\boldsymbol{\delta}} b^4}{\in}\left\{(1+K)b+\dfrac{1}{K_1}\right\} - \dfrac{Kb^4}{\in}}{a^2\left(b+2A-\dfrac{A\bar{\boldsymbol{\delta}}b}{\in}\right)^2}$$

$$+\frac{\begin{array}{l}\dfrac{P_m k_x^2 Qb}{\in P_r}\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]\\ \times\left[\dfrac{\bar{\boldsymbol{\delta}} b^3}{\in}\left(b+\dfrac{1}{K_1}\right)+\dfrac{\bar{\boldsymbol{\delta}} b^4 K}{\in}+\dfrac{\bar{\boldsymbol{\delta}} b^2 P_m k_x^2 Q}{\in^2 P_r}-b^3K\right]-\dfrac{P_m k_x^2 Qb}{\in P_r}\left(b+2A-\dfrac{A\bar{\boldsymbol{\delta}}b}{\in}\right)\\ \times\left[b\left(b+\dfrac{1}{K_1}\right)(b+2A)+b^2K(b+A)+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]\\ \times\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\left\{2\left(b+\dfrac{1}{K_1}\right)+bK\right\}+\dfrac{2P_m k_x^2 Q}{\in P_r}\right]\end{array}}{a^2\left(b+2A-\dfrac{A\bar{\boldsymbol{\delta}}b}{\in}\right)^2\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]^2}$$

...(50)

Clearly, $\dfrac{dR}{dA} > 0$ when $\bar{\boldsymbol{\delta}} > \max.\left\{K, \dfrac{\in}{A}+\dfrac{2\in}{b}\right\}$. Thus, the micropolar coefficient has stabilizing effect

when $\bar{\boldsymbol{\delta}} > \max.\left\{K, \dfrac{\in}{A}+\dfrac{2\in}{b}\right\}$

In the absence of Hall current $(\beta_e = 0)$ and magnetic field $(Q = 0)$, the equation (50) reduces to

$$\frac{dR}{dA} = \frac{\dfrac{\bar{\boldsymbol{\delta}} b^4 (1+K)}{\in} + \dfrac{b^4}{\in}(\bar{\boldsymbol{\delta}} - K)}{a^2\left(b+2A-\dfrac{A\bar{\boldsymbol{\delta}}b}{\in}\right)^2} \qquad ...(51)$$

It is clear that $\dfrac{dR}{dA} > 0$ when $\bar{\boldsymbol{\delta}} > K$, thus the micropolar coefficient A has stabilizing effect when $\bar{\boldsymbol{\delta}} > K$.

In the absence of Hall current, magnetic field and micropolar heat conduction parameter $(\bar{\boldsymbol{\delta}} = 0)$, (51) reduces to $\dfrac{dR}{dA} = \dfrac{-K}{a^2 (b+2A)^2}$, which is clearly negative. Thus, in the absence of Hall current, magnetic field and micropolar heat conduction parameter, the micropolar coefficient has destabilizing effect.

From (48), we have

$$\frac{dR}{d\bar{\delta}} = \frac{Ab/\in}{a^2\left(b+2A-\dfrac{A\bar{\boldsymbol{\delta}}b}{\in}\right)^2}\left[\frac{\begin{array}{l}\left[b^2\left\{(1+K)b+\dfrac{1}{K_1}\right\}(b+2A)-\dfrac{KAb^3}{\in}\right]\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\right.\\ \left.\times\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]+\dfrac{P_m k_x^2 Qb}{\in P_r}\\ \times\left[b\left(b+\dfrac{1}{K_1}\right)(b+2A)+b^2K(b+A)+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]\end{array}}{\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]}\right]$$

Clearly $\dfrac{dR}{d\bar{\delta}}$ is positive when $\in > \dfrac{1}{2}$. Thus, the micropolar heat conduction parameter $\bar{\boldsymbol{\delta}}$ has a stabilizing effect when $\in > \dfrac{1}{2}$.

From (48), we have

$$\frac{dR}{dQ} = \frac{\begin{array}{l}\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\left(\dfrac{P_m k_x^2 b^2}{\in P_r}\right)\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}^2\right.\\ +\dfrac{2QP_m^2 k_x^4 b(b+2A)}{\in^2 P_r^2}\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}\\ \left.+\dfrac{P_m^3 k_x^6 Q^2 b(b+2A)^2}{\in^3 P_r^3}\right]\end{array}}{a^2\left(b+2A-\dfrac{A\bar{\delta}b}{\in}\right)\left[\left(b+\dfrac{P_m^2 k_x^2 \beta_e}{P_r^2}\right)\left\{\left(b+\dfrac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}+\dfrac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]^2}$$

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Clearly $\frac{dR}{dQ}>0$ when $\bar{\boldsymbol{\delta}}<\frac{\in}{A}$ and $\frac{\partial R}{\partial Q}<0$ when $\bar{\boldsymbol{\delta}}>\frac{\in}{A}+\frac{2\in}{b}$. Thus, the magnetic field has stabilizing effect when $\bar{\boldsymbol{\delta}}<\frac{\in}{A}$ and it will have destabilizing effect when $\bar{\boldsymbol{\delta}}>\frac{\in}{A}+\frac{2\in}{b}$.

From (48), we have

$$\frac{dR}{d\beta_e}=\frac{-\frac{P_m^2 k_x^2}{P_r^2}\left[\left(b+\frac{1}{K_1}\right)(b+2A)+bK(b+A)\right]\frac{P_m k_x^2 Qb}{\in P_r}\times\left[b\left(b+\frac{1}{K_1}\right)(b+2A)+b^2K(b+A)+\frac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]}{a^2\left(b+2A-\frac{A\bar{\boldsymbol{\delta}}b}{\in}\right)\left[\left(b+\frac{P_m^2 k_x^2\beta_e}{P_r^2}\right)\left\{\left(b+\frac{1}{K_1}\right)(b+2A)+bK(b+A)\right\}+\frac{P_m k_x^2 Q(b+2A)}{\in P_r}\right]^2}$$

Clearly $\frac{dR}{d\beta_e}<0$ when $\bar{\boldsymbol{\delta}}<\frac{\in}{A}$ and $\frac{dR}{d\beta_e}>0$ when $\bar{\boldsymbol{\delta}}>\frac{\in}{A}\left(\frac{1+2A}{b}\right)$. Thus, the Hall current has destabilizing effect when $\bar{\boldsymbol{\delta}}<\frac{\in}{A}$ and it will have stabilizing effect when $\bar{\boldsymbol{\delta}}>\frac{\in}{A}\left(1+\frac{2A}{b}\right)$

In the absence of micropolar heat conduction parameter $(\bar{\boldsymbol{\delta}}=0)$, above equation reduces to $\frac{dR}{d\beta_e}<0$ Thus, in the absence of micropolar heat conduction parameter, the Hall current has a destabilizing effect.

IX. OSCILLATORY CONVECTION

In order to investigate oscillatory modes we follow Chandrasekhar, put $\sigma=i\sigma_i$ in (46) and separating real and imaginary parts, the real part gives

$$Ra^2=\frac{b_1\sigma_i^6+b_2\sigma_i^4+b_3\sigma_i^2+b_4}{a_1\sigma_i^4+a_2\sigma_i^2+a_3} \qquad ...(52)$$

Where $a_1=-\left\{\bar{\mathbf{j}}\in^2 P_r^2\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)+\frac{2\in P_r^2\bar{\mathbf{j}}b^2}{P_m}+\in P_r^2\bar{\mathbf{j}}b\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\right\}$

$$a_2=\frac{\in^2 P_r^2\bar{\mathbf{j}}b^2}{P_m^2}\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)+\frac{2\in^2 P_r^2\bar{\mathbf{j}}b}{P_m}(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)$$

$$-\frac{2K^2\in^2 P_r^2 b^2\bar{\mathbf{j}}}{P_m}+k_x^2 Q\bar{\mathbf{j}}\in P_r\left(\frac{\bar{\mathbf{j}}b}{P_m}+Cb+2K\right)+b\in^2 k_x^2\beta_e\bar{\mathbf{j}}\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{\in_1}\right)\right)$$

$$+\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\frac{\in P_r^2\bar{\mathbf{j}}b^3}{P_m^2}+\in^2 P_r^2 b\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)$$

$$+\frac{2\in^2 P_r^2 b^2}{P_m}\left(Cb-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)$$

$$-\in^2 P_r^2 K^2 b^2\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)+k_x^2 Q\bar{\mathbf{j}}.\in P_r b\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)+b^2\in k_x^2\beta_e\bar{\mathbf{j}}\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)$$

$$a_3=-\frac{\in^2 P_r^2 b^3}{P_m^2}\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)+\frac{\in^2 P_r^2 K^2 b^2}{P_m^2}\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)$$

$$-\frac{\in P_r k_x^2 Q b^2}{P_m}\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)(Cb+2K)-b^2\in^2 k_x^2\beta_e\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)$$

$$+K^2\in^2 k_x^2\beta_e b^3\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)$$

$$b_1=\frac{bE_r\bar{\mathbf{j}}}{\in}\left\{\in^2 P_r^2\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)+\frac{2\in P_r^2\bar{\mathbf{j}}}{P_m}\right\}$$

$$+\in P_r^2\bar{\mathbf{j}}\left\{\frac{\bar{\mathbf{j}}b^2}{\in}+bEP_r\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)\right\}$$

$$b_2=\frac{-bEP_r\bar{\mathbf{j}}}{\in}\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)+\frac{2\in^2 P_r^2 b}{P_m}(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)\right.$$

$$\left.-\frac{2K^2\in^2 P_r^2 b^2}{P_m}+k_x^2 Q\in P_r\left(\frac{\bar{\mathbf{j}}b}{P_m}+Cb+2K\right)+b\in^2 k_x^2\beta_e\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)\right\}$$

$$-\left\{bEP_r(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)+b^2\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)-\frac{K^2b^2EP_r}{\in}\right\}$$

$$\times\left\{\in^2 P_r^2\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)-\frac{2\in P_r^2 b\bar{\mathbf{j}}}{P_m}\right\}$$

$$-\in P_r^2\bar{\mathbf{j}}\left(b^2(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)-\frac{K^2b^3}{\in}\right)-\left\{\frac{\bar{\mathbf{j}}b^2}{\in}+bEP_r\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)\right\}$$

$$\times\left\{\frac{\in P_r^2 b^2\bar{\mathbf{j}}}{P_m^2}+\in^2 P_r^2(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)+\frac{2\in^2 P_r^2 b}{P_m}\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)\right.$$

$$\left.-K^2\in^2 P_r^2 b+k_x^2 Q\bar{\mathbf{j}}\in P_r+b\in k_x^2\beta_e\bar{\mathbf{j}}\right\}-k_x^2 Qb\left\{\frac{P_r^2 b\bar{\mathbf{j}}^2E}{P_m}\right.$$

$$\left.+\in E P_r^2\bar{\mathbf{j}}\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)+P_r\bar{\mathbf{j}}\left(EP_r(Cb+2K)+b\bar{\mathbf{j}}\right)\right\}$$

$$b_3=\left\{bEP_r(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)+b^2\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)-\frac{K^2b^2EP_r}{\in}\right\}$$

$$\times\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)+\frac{2\in^2 P_r^2 b}{P_m}(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)\right.$$

$$\left.-\frac{2K^2\in^2 P_r^2 b^2}{P_m}+k_x^2 Q\in P_r\left(\frac{\bar{\mathbf{j}}b}{P_m}+Cb+2K\right)+b\in^2 k_x^2\beta_e\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)\right\}$$

$$+\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}(Cb+2K)\left(b+bK+\frac{b}{K_1}\right)-\frac{K^2\in^2 P_r^2 b^3}{P_m^2}+\frac{k_x^2 Q\in P_r b(Cb+2K)}{P_m}\right.$$

$$\left.+k_x^2 Qb\left\{\frac{\in EP_r^2\bar{\mathbf{j}}b}{P_m}(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)+\frac{P_r\bar{\mathbf{j}}b^2}{P_m}(Cb+2K)\right.\right.$$

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$$+\in P_r b(Cb+2K)\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)-\frac{K^2\in EP_r^2\,\bar{\mathbf{j}}\,b^2}{P_m}+k_x^2 Q\,\bar{\mathbf{j}}\,EP_r(Cb+2K)$$

$$+\in P_r(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)\left(EP_r(Cb+2K)+b\,\bar{\mathbf{j}}\right)+\frac{\in P_r b}{P_m}\left(EP_r(Cb+2K)+b\,\bar{\mathbf{j}}\right)$$

$$\left.\times\left(\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)\right)-K^2 b\in P_r\left(EP_r(Cb+2K)+b\bar{\mathbf{j}}\right)+k_x^2 Q\bar{\mathbf{j}}\left(EP_r(Cb+2K)+bj\right)\right\}$$

$$b_4=\left\{b^2(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)-\frac{K^2b^3}{\in}\right\}.\left\{\frac{\in^2 P_r^2 b^2}{P_m^2}(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)\right.$$

$$\left.-\frac{K^2\in^2 P_r^2 b^3}{P_m^2}+\frac{k_x^2 Q\in P_r b(Cb+2K)}{P_m}+b\in^2 k_x^2\beta_e(Cb+2K)\left(b+bK+\frac{1}{K_1}\right)-K^2b^2\in^2 k_x^2\beta_e\right\}$$

$$-k_x^2 Q\left\{\frac{\in P_r b^2}{P_m}(Cb+2K)^2\left(b+bK+\frac{1}{K_1}\right)-\frac{K^2\in P_r b^3}{P_m}(Cb+2K)+k_x^2 Qb(Cb+2K)^2\right\}$$

and imaginary part gives

$$Ra^2=\frac{b_1'\sigma_i^6+b_2'\sigma_i^4+b_3'\sigma_i^2+b_4'}{a_1'\sigma_i^4+a_2'\sigma_i^2+a_3'} \quad ...(53)$$

where $a_1'=\in P_r^2\,\bar{\mathbf{j}}^2$, $b_1'=-bEP_r^3\,\bar{\mathbf{j}}^2$ and rest a_2', a_3' and b_2', b_3', b_4' can be taken from the above equation.

Eliminating R between (52) and (53), we have

$$f_0 s^5+f_1 s^4+f_2 s^3+f_4 s^2+f_5 s+f_6=0 \quad ...(54)$$

Where $s=\sigma_i^2$ and

$$f_0=b_1 a_1'-b_1' a_1$$

$$=b^2\left\{\in P_r^4\,\bar{\mathbf{j}}^4+EP_r^5\,\bar{\mathbf{j}}^3 K\bar{\boldsymbol{\delta}}+E\in^2 P_r^5\bar{\mathbf{j}}^4(1+K)\right\}$$

$$+b\left\{2KE\in P_r^5\,\bar{\mathbf{j}}^3+\frac{E\in^2 P_r^5\,\bar{\mathbf{j}}^4}{K_1}\right\}$$

$$\Rightarrow \qquad f_0>0$$

Let

$$\frac{Cb+2K}{\in}+\bar{\mathbf{j}}\left(b+bK+\frac{1}{K_1}\right)=L_1,\quad (Cb+2K)\left(b+bK+\frac{1}{K_1}\right)=L_2$$

then

$$f_1=\frac{2\in EP_r^5\,\bar{\mathbf{j}}^3 b^4}{P_m^2}\left(1-\frac{\bar{\mathbf{j}}}{P_m}\right)+2\in EP_r^4\,\bar{\mathbf{j}}^3 b^2 k_x^2 Q\left(1-\frac{\bar{\mathbf{j}}}{P_m}\right)$$

$$+\in P_r^3\,\bar{\mathbf{j}}^3 b^2 k_x^2 Q\left(2EP_r\left(C-\frac{k\bar{\boldsymbol{\delta}}}{\in}\right)-\bar{\mathbf{j}}\right)+\in^2 EP_r^4\,\bar{\mathbf{j}}^3 k_x^2 QbL_1(\in^2 P_r^2 L_1-1)$$

$$+\in^2 EP_r^3 b^2\,\bar{\mathbf{j}}^2 L_1\left(K^2\in P_r^2-\bar{\mathbf{j}}k_x^2\beta_e\right)+\in P_r^2\,\bar{\mathbf{j}}^3 b^3 k_x^2\beta_e(2EP_r-\in\bar{\mathbf{j}})$$

$$+\frac{2\in EP_r^3\,\bar{\mathbf{j}}^3 b^3}{P_m}(K^2\in P_r^2-\bar{\mathbf{j}}k_x^2\beta_e)+2\in EP_r^3\,\bar{\mathbf{j}}^2 b^3\left(C-\frac{K\delta}{\in}\right)\left(\bar{\mathbf{j}}k_x^2\beta_e-K^2\in P_r^2\right)$$

$$+K^2\in^2 P_r^4 b^3\,\bar{\mathbf{j}}^2(\bar{\mathbf{j}}-2EP_r)+b\in^3 EP_r^5\,\bar{\mathbf{j}}^2 L_1 L_2(1-2\bar{\mathbf{j}})+2\in^3 P_r^4\,\bar{\mathbf{j}}\,b^2 L_1^2\left(\bar{\mathbf{j}}-\frac{2EP_r}{P_m}\right)$$

$$+2\in^3 EP_r^5\,\bar{\mathbf{j}}\,b^2 L_1^2\left(\frac{3\bar{\mathbf{j}}}{P_m}-\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\right)+2\in^2 EP_r^5\,\bar{j}^2 b^2 L_2\left(1-\frac{\bar{\mathbf{j}}^2}{P_m}\right)$$

$$+2\in^2 P_r^4\,\bar{\mathbf{j}}^2 b^3 L_1\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\left(\in-\frac{EP_r}{P_m}\right)+\frac{2\in P_r^4\,\bar{\mathbf{j}}^3 b^4}{P_m}\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\left(\in-\frac{EP_r}{P_m}\right)$$

$$+\in^2 P_r^4\,\bar{\mathbf{j}}^2 b^2 L_2\left(2EP_r\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)-\bar{\mathbf{j}}^2\right)+\frac{\in^2 P_r^4\,\bar{\mathbf{j}}^2 b^3 L_1}{P_m}\left(\frac{3EP_r\,\bar{\mathbf{j}}}{P_m}-2\right)$$

$$+\in^2 P_r^4\,\bar{\mathbf{j}}^2 b^3 L_1\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\left(\frac{2\in EP_r}{P_m}-1\right)+\frac{\in P_r^4\,\bar{\mathbf{j}}^3 b^4}{P_m}\left(\frac{3\bar{\mathbf{j}}}{P_m}-2\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)\right)$$

$$+\in^3 EP_r^5\,\bar{\mathbf{j}}^2 L_1 L_2+\in^4 EP_r^5\,\bar{\mathbf{j}}\,bL_1^3+\frac{2\in^2 P_r^4\,\bar{\mathbf{j}}^3 b^2 L_1}{P_m}+\frac{2\in^3 EP_r^6\,\bar{\mathbf{j}}^4 b^2 k_x^2 QL_1}{P_m}$$

$$+\frac{2\in^2 P_r^5\,\bar{\mathbf{j}}^3 bL_2}{P_m}+\frac{4\in^2 P_r^4\,\bar{\mathbf{j}}^3 b^3 L_1}{P_m}+\frac{2\in^3 EP_r^6\,\bar{\mathbf{j}}^4 k_x^2 Qb^2 L_1}{P_m}+\frac{4\in^2 EP_r^6\,\bar{\mathbf{j}}^5 b^3 k_x^2 Q}{P_m^2}$$

$$+\in^3 EP_r^5\,\bar{\mathbf{j}}^2 bL_2\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)+\in^4 EP_r^5\,\bar{\mathbf{j}}\,b^2 L_1^2\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)$$

$$+\in^4 EP_r^6\,\bar{\mathbf{j}}^3 b^2 k_x^2 QL_1\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)+\frac{2\in^2 EP_r^6\,\bar{\mathbf{j}}^4 b^3 k_x^2 Q}{P_m}\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)$$

Now $f_1>0 \Rightarrow$

$$\left.\begin{array}{l}\left(\frac{C}{K}-\frac{7}{4K}\right)<\frac{\bar{\boldsymbol{\delta}}}{\in}<\min\left\{\frac{C}{K},\left(\frac{C}{K}-\frac{1}{KP_m}\right)\right\},\quad \frac{P_m}{2\in E}<P_r<\frac{\in P_m}{E},\\ \left[K_1<\in^2 P_r^2\bar{\mathbf{j}}\text{ or }K>\frac{1}{2\in P_r^2}\right]\\ \max.\left[2EP_r,\frac{2EP_r}{P_m},\frac{2P_m}{3EP_r}\right]<\bar{\mathbf{j}}<\min\left\{\frac{1}{2},P_m,\sqrt{P_m},\frac{2EP_r}{\in},\sqrt{2EP_r\left(C-\frac{K\bar{\boldsymbol{\delta}}}{\in}\right)}\right\}\end{array}\right\}$$

...(55)

and $\quad \beta_e<\dfrac{K^2\in P_r^2}{\bar{\mathbf{j}}k_x^2}$

Since $s=\sigma_i^2$ so that the sum of the roots of equation (54) must be positive but the sum of its roots is $\left(-\frac{f_0}{f_1}\right)$, therefore the sufficient conditions for non-existence of oscillatory modes are given by $f_0>0$ and $f_1>0$

But $f_0>0$ and $f_1>0\Rightarrow$ Condition (55) including

$$\beta_e<\frac{K^2\in P_r^2}{\bar{\mathbf{j}}k_x^2}$$

Hence, the sufficient conditions for non-existence of oscillatory modes are the given by (55).

X. OBSERVATIONS

In stationary convection, the variations of the critical thermal Rayleigh number R with respect to the variations of Micropolar heat conduction parameter $(\bar{\delta})$ and Hall parameter (β_e), have been predicted by the following graphs respectively

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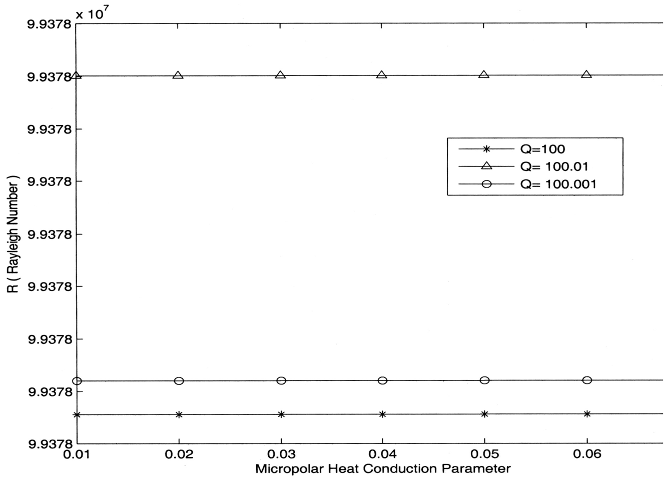

Fig.2 : Marginal instability curve for the variation of R vs $\bar{\delta}$ for A=0.5, $\in$=0.6, β_e =0.2, P_r=2, P_m=4, K=1, K_1=0.5, a=1, k_x=0.05.

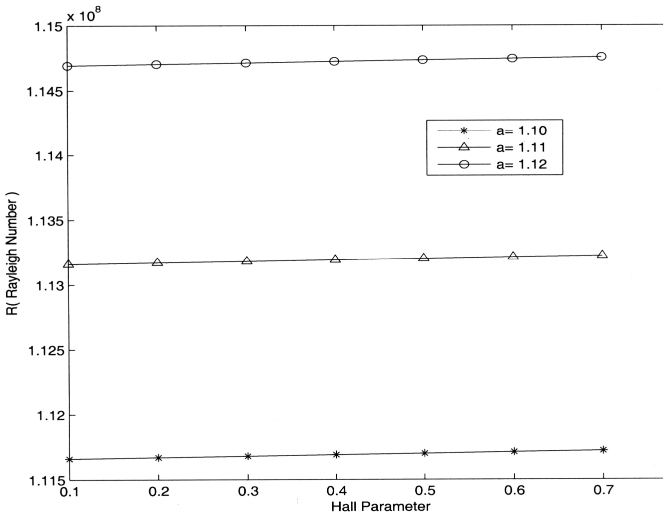

Fig. 3 : Marginal instability curve for the variation of R vs β_e for A=0.5, $\in$=0.5, P_r=2, P_m=4, K=1, k_x=0.05, K_1=0.5, $\bar{\delta}$ =1.2, Q=10.

XI. CONCLUSIONS

For Stationary Convection:-

1. When $\bar{\delta} < \dfrac{\in}{A}$ and $\beta_e < \dfrac{2Ab\,P_r^2}{P_m^2\,k_x^2}$, the critical Rayleigh number increases as the medium permeability decreases, thus, the medium permeability has a destabilizing effect under the above conditions. In the absence of Hall current, the medium permeability has destabilizing effect if $\bar{\boldsymbol{\delta}} < \dfrac{\in}{A}$.
2. When $\bar{\boldsymbol{\delta}} < \dfrac{\in}{A}$ and $0 < \in < \dfrac{1}{2}$, the critical Rayleigh number increases as the coupling parameter decreases, thus, the coupling parameter has destabilizing effect. When $\dfrac{1}{2} < \in < 1$ and $\bar{\boldsymbol{\delta}} < \dfrac{b}{2}$, the critical Rayleigh number increases as the coupling parameter increases, thus under this condition the coupling parameter has stabilizing effect.
3. When $\bar{\boldsymbol{\delta}} > \max.\left\{K, \dfrac{2\in}{b}, \left(\dfrac{\in}{A} + \dfrac{2\in}{b}\right)\right\}$, the micropolar coefficient has stabilizing effect**(see fig.2**). In the absence of Hall current, magnetic field and micropolar heat condition parameter, the micropolar coefficient has destabilising effect.
4. When $\bar{\boldsymbol{\delta}} < \dfrac{\in}{A}$, the magnetic field has stabilizing effect and when $\bar{\boldsymbol{\delta}} > \left(\dfrac{\in}{A} + \dfrac{2\in}{b}\right)$, the magnetic field has destabilizing effect.
5. When $\bar{\boldsymbol{\delta}} < \dfrac{\in}{A}$, the Hall current has destabilizing effect.
6. When $\bar{\boldsymbol{\delta}} > \dfrac{\in}{A}\left(1 + \dfrac{2A}{b}\right)$, the Hall current has stabilizing effect(**see fig.3**).
7. In the absence of micropolar heat conduction parameter, the Hall current has a destabilizing effect.
8. The micropolar heat conduction parameter has a stabilizing effect whatever the strength of magnetic field is applied.

For Oscillatory Convection:

The sufficient conditions for non-existence of oscillatory modes are

$$\left(\frac{C}{K} - \frac{7}{4K}\right) < \frac{\bar{\boldsymbol{\delta}}}{\in} < \min\left\{\frac{C}{K}, \left(\frac{C}{K} - \frac{1}{KP_m}\right)\right\}, \quad \frac{1}{2\in E} < \frac{P_r}{P_m} < \frac{\in}{E},$$

$$\left[K_1 < \in^2 P_r^2\,\bar{\mathbf{j}} \text{ or } K > \frac{1}{2\in P_r^2}\right]$$

$$\max.\left\{2EP_r, \frac{2EP_r}{P_m}, \frac{2P_m}{3EP_r}\right\} < \bar{\mathbf{j}} < \min.\left\{\frac{1}{2}, P_m, \sqrt{P_m}, \frac{2EP_r}{\in}, \sqrt{2EP_r\left(C - \frac{K\bar{\delta}}{\in}\right)}\right\}$$

and $\beta_e < \dfrac{K^2 \in P_r^2}{\bar{j}\,k_x^2}$, provided $C > \dfrac{K\bar{\delta}}{\in}$

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