Geometric Nonlinear Analysis of Laminated Composite Plates Using Finite Element Method

Dipali B. Patil, Rajan L. Wankhade, P. K. Deshpande

Abstract— Finite Element Analysis for geometrically nonlinear behavior of laminated composite plates is presented and compared with the reported investigations. A first order displacement field that accounts for transverse shear effects under geometric nonlinear condition is employed in the formulation of a four node, rectangular, element with five degrees of freedom per node. The formulation demonstrates its excellence in the performance for predicting response at various lay ups and plies conditions.

Index Terms—Finite element method, plates and shells, geometric non linear analysis, composites.

I. INTRODUCTION

Thin and laminated plates/panels are one of the major load bearing structural elements in high performance structures. Composite laminates have many applications as advanced engineering materials, primarily as components in aircrafts, power plants, civil engineering structures, ships, cars, rail vehicles, robots, prosthetic devices etc. The major advantage of composite material is ability of the controllability fiber alignment. By arranging layers & fiber direction of laminates it can gain required strength & stiffness to meet specific design conditions. Most of the impact problems have been formulated using the small deflection theory which is adequate if the impact load is small. Theory regarding plates and shells and finite element method is given by Timoshenko, Zienkiewicz and Krishnamoorthy respectively. Ghugal and shimpi (2002) presented review of refined shear deformation theories of isotropic and anisotropic laminated plates. Wankhade (2011) analyzed skew plates for geometric nonlinear analysis using finite element method. Chaudhari (2011) also studied geometric nonlinear analysis of composite plates. Akavci (2007) used first order shear deformation theory for symmetrically laminated composite plates on elastic foundation. In the present study, a finite element including the effect of geometric non-linearity is employed in the impact analysis of laminated composite plates. A series of numerical examples are presented that provide insight into certain interesting behavior of laminated composite plates. Results are compared to available literature results & other available sources of numerical studies to validate the element.

II. FINITE ELEMENT METHODOLOGY

Fig. 2.1 shows a typical plan of laminated plate consisting of orientation of fibers with different axes

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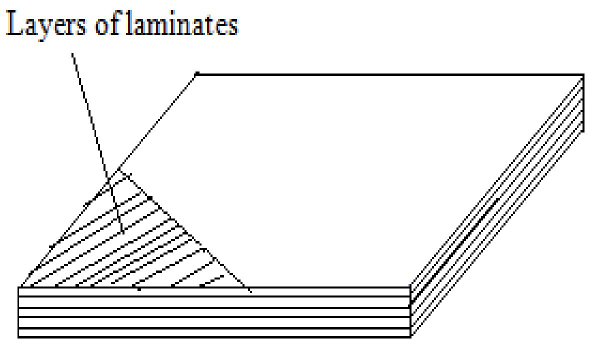

Fig. 2.1 Laminated composite plate provided with fiber orientation

At each node equal degrees of freedom is provided i.e. u, v, w, ϕ_x, ϕ_y

And hence for FOST writing,

$$u = u_0 + z\phi_x$$
$$v = v_0 + z\phi_y$$
$$w = w_0 \tag{2.1}$$

The coordinate x and y of any point within the element can be obtained by the four geometry nodes on the boundary of the plate expressed as,

$$x = \sum_{i=1}^{4} N_i x_i$$
$$y = \sum_{i=1}^{4} N_i y_i \tag{2.2}$$

Where, Ni's are Lagrangian isoparametric shape functions and are given by,

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \tag{2.3}$$

Displacement within element domain can be given as,

$$u = \sum_{i=1}^{4} N_i u_i$$
$$v = \sum_{i=1}^{4} N_i v_i$$
$$w = \sum_{i=1}^{4} N_i w_i \tag{2.4}$$

Formulation for linear analysis

In case of linear analysis, plate observes following three types of strains. These strains are linearly related with displacements and are given as following,

A. Middle plane membrane strains

$$\underbrace{\varepsilon_p^L}_{3\times1} = \begin{Bmatrix} \varepsilon_{xp} \\ \varepsilon_{yp} \\ \gamma_{xyp} \end{Bmatrix} \tag{2.5}$$

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B. Curvature strains/Bending strains

Curvature strains are linearly related to bending displacement as

$$\underbrace{\varepsilon_b^L}_{3\times1} = \begin{Bmatrix} \varepsilon_{xxb} \\ \varepsilon_{yyb} \\ \gamma_{xyb} \end{Bmatrix} \tag{2.6}$$

$$= z.K \tag{2.7}$$

C. Shear strains

$$\underbrace{\gamma}_{2\times1} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{2.8}$$

Thus combining them together,

$$\begin{Bmatrix} \underbrace{\varepsilon}_{6\times1} \end{Bmatrix} = \begin{Bmatrix} \underbrace{\varepsilon_p}_{3\times1} \\ \underbrace{\varepsilon_b}_{3\times1} \end{Bmatrix} \tag{2.9}$$

And $$\underbrace{\gamma}_{2\times1} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{2.10}$$

Where, $\varepsilon_p, \varepsilon_b$ *and* γ are membrane, bending and shear components of strains respectively.

Strain-Displacement Matrix

Strains are related with displacements as follows,

$$\underbrace{\varepsilon}_{6\times1} = \underset{6\times20}{B} \underbrace{\delta_e}_{20\times1} \tag{2.11}$$

$$\underbrace{\gamma}_{2\times1} = \underset{2\times20}{B_s} \underbrace{\delta_e}_{20\times1} \tag{2.12}$$

where Bm and Bs are strain displacement matrix which contains derivatives of shape functions.

$[B]_m$ – contribution due to membrane and bending effect

$[B]_s$ – contribution due to transverse shear

Geometric non-linear approach

In linear analysis, it is assumed that both displacements and strains developed in the structure are small. In practical terms this means that geometry of the element remains basically unchanged during loading process and hence linear strain assumptions can be used. In practice such assumptions fails frequently even though actual strains may be small and elastic limit of ordinary structural materials not exceeded. If accurate determination of the displacement is needed for plates with large deflections analysis, geometric non-linearity may have to be considered. Whether the displacement or strains are large or small, equilibrium conditions between internal and external forces have to be satisfied. If $\Psi(a)$ is a sum of internal and external geometric forces,

$$\Psi(a) = \int_v \bar{B}^T \sigma \, dv - f = 0 \tag{2.13}$$

In which $\bar{B}$ is defined from $d\varepsilon = \bar{B}\, da$. $\bar{B}$ contains linear as well as non-linear strain displacement relation.

And $$\bar{B} = B_0 + B_{NL} \tag{2.14}$$

In which B_0 is the same matrix as in linear analysis. And B_{NL} contains some of the non-linear terms. A general definition of strains which is valid whether displacements or strains are large or small is,

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left[\frac{\partial u}{\partial x}.\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y}\right] \tag{2.15}$$

and similar expressions for other strains. These formulae are defined by Green and saint Venant (Zienkiewicz, O. C. (1989)).

$$\varepsilon_b^L = \begin{Bmatrix} \varepsilon_{xxb} \\ \varepsilon_{yyb} \\ \varepsilon_{xyb} \end{Bmatrix} \tag{2.16}$$

For large deflection analysis of plates the non-linear expressions given in equation 2.17 reduce to von Karman equations. The non-linear strains at any point within plate are given by,

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left(\frac{\partial w}{\partial x}.\frac{\partial w}{\partial y}\right)$$

$$\underbrace{\gamma}_{2\times1} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \phi x + \frac{\partial w}{\partial x} \\ \phi y + \frac{\partial w}{\partial y} \end{Bmatrix} \tag{2.17}$$

Strains

In case of non-linear analysis, plate observes some of the non-linear terms in the membrane and bending strains. As these non-linear terms relate membrane and bending displacements and strains and are unaffected by shear, shear can be decoupled and its formulation can be separated from membrane-bending behavior. Hence these strains are given as following,

$$\underbrace{\varepsilon_p}_{3\times1} = \varepsilon_p^L + \varepsilon_p^N \tag{2.18}$$

in which ε_p^L and ε_p^N are linear and non-linear components of middle plane membrane strains and combining linear and non-linear terms membrane strains are given as,

Curvature strains/Bending strains

Curvature strains are linearly related to bending displacement as

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$$= z \begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \end{Bmatrix}$$

$$= z.K \tag{2.19}$$

c. Shear strains

$$\underbrace{\gamma}_{2\times1} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

$$= \begin{Bmatrix} \phi_x + \dfrac{\partial w}{\partial x} \\ \phi_y + \dfrac{\partial w}{\partial y} \end{Bmatrix} \tag{2.20}$$

$$\left\{ \underbrace{\varepsilon}_{6\times1} \right\} = \begin{Bmatrix} \underbrace{\varepsilon_p}_{3\times1} \\ \underbrace{\varepsilon_b}_{3\times1} \end{Bmatrix}$$

Thus

$$= \begin{Bmatrix} \varepsilon_p^L \\ \varepsilon_b^L \end{Bmatrix} + \begin{Bmatrix} \varepsilon_p^N \\ 0 \end{Bmatrix} \tag{2.21}$$

Hence,

$$\underbrace{\varepsilon}_{6\times1} = \underbrace{\varepsilon}_{6\times1}^{L} + \underbrace{\varepsilon}_{6\times1}^{N} \tag{2.22}$$

and shear strains can separately be written as,

$$\underbrace{\gamma}_{2\times1} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{2.23}$$

In which $\{\varepsilon_p\},\{\varepsilon_b\}\,and\,\{\gamma\}$ are membrane, bending and shear components of strains respectively. $\underset{6\times1}{\varepsilon}$ is combined strain vector of membrane and bending strains. $\underset{2\times1}{\gamma}$ is a vector containing shear strains. Subscript 'p' stands for in-plane, 'b' for bending, 'L' for linear and subscript 'N' stands for non-linear.

Displacement-strain relation

Strains are related with displacements as follows,

$$\underbrace{\varepsilon}_{6\times1} = \underset{6\times20}{B} \underbrace{\delta_e}_{20\times1} \tag{2.24}$$

$$\begin{Bmatrix} \varepsilon_p \\ \varepsilon_b \end{Bmatrix} = \begin{bmatrix} B_p^L & B_b^N \\ 0 & B_b^L \end{bmatrix} \begin{Bmatrix} \delta_p \\ \delta_b \end{Bmatrix}$$

also,

$$\underbrace{\gamma}_{2\times1} = \underset{2\times20}{B_s} \underbrace{\delta_e}_{20\times1} \tag{2.25}$$

Change in strain can be related to change in displacement as,

$$\underbrace{\delta\varepsilon}_{6\times1} = \underset{6\times20}{\bar{B}} \underbrace{d\delta_e}_{20\times1} \tag{2.26}$$

Also,

$$\begin{Bmatrix} d\varepsilon_p \\ d\varepsilon_b \end{Bmatrix} = \begin{bmatrix} B_p^L & \bar{B}_b^N \\ 0 & B_b^L \end{bmatrix} \begin{Bmatrix} d\delta_p \\ d\delta_b \end{Bmatrix} \tag{2.27}$$

Matrices [B] and $[\bar{B}]$ are different here. Various components of strains and change in strains are written as,

$$\begin{aligned} \underbrace{\varepsilon_p^L}_{3\times1} &= \underset{3\times8}{B_p^L} \underbrace{\delta_p}_{8\times1} \\ \underbrace{\varepsilon_b^L}_{3\times1} &= \underset{3\times12}{B_b^L} \underbrace{\delta_b}_{12\times1} \\ \underbrace{\varepsilon_p^N}_{3\times1} &= \underset{3\times12}{B_b^N} \underbrace{\delta_b}_{12\times1} \end{aligned} \tag{2.28}$$

Change in strains,

$$\begin{aligned} \underbrace{d\varepsilon_p^L}_{3\times1} &= \underset{3\times8}{B_p^L} \underbrace{d\delta_p}_{8\times1} \\ \underbrace{d\varepsilon_b^L}_{3\times1} &= \underset{3\times12}{B_b^L} \underbrace{d\delta_b}_{12\times1} \\ \underbrace{d\varepsilon_p^N}_{3\times1} &= \underset{3\times12}{\bar{B}_b^N} \underbrace{d\delta_b}_{12\times1} \end{aligned} \tag{2.29}$$

Matrices $\underset{3\times12}{B_b^N}$ and $\underset{3\times12}{\bar{B}_b^N}$ are different due to geometric non-linear behavior of structure. The non-linear components of middle plane strains are written as,

$$\underbrace{\varepsilon_p^N}_{3\times1} = \frac{1}{2} \begin{Bmatrix} \dfrac{\partial w}{\partial x}^2 \\ \dfrac{\partial w}{\partial y}^2 \\ 2\dfrac{\partial w}{\partial x}.\dfrac{\partial w}{\partial y} \end{Bmatrix} = \frac{1}{2} \begin{bmatrix} \dfrac{\partial w}{\partial x} & 0 \\ 0 & \dfrac{\partial w}{\partial y} \\ \dfrac{\partial w}{\partial y} & \dfrac{\partial w}{\partial x} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial w}{\partial x} \\ \dfrac{\partial w}{\partial y} \end{Bmatrix}$$

$$= \frac{1}{2} \underset{3\times2}{[A]} \underset{2\times12}{[G]} \underset{12\times1}{\{\delta_b\}} \tag{2.30}$$

Hence

$$\underset{3\times12}{B_b^N} = \frac{1}{2} \underset{3\times2}{[A]} \underset{2\times12}{[G]} \tag{2.31}$$

Here [A] depends on the displacement vector of current deformed configuration of structure and slope matrix {θ} depends on next deformed configuration of structure which is to be calculated. Matrix [G] is purely based on derivatives of shape functions and coordinates.

$$d\left\{\varepsilon_p^N\right\} = d\left(\frac{1}{2}[A][\theta]\right)$$

$$= [A]\{d\theta\}$$

i.e.

$$\left\{d\varepsilon_p^N\right\} = \underset{3\times1}{[A]} \underset{3\times2}{[G]} \underset{12\times1}{\{d\delta_b\}} \tag{2.32}$$

Hence

$$\underset{3\times12}{\left[\bar{B}_b^N\right]} = \underset{3\times2}{[A]} \underset{2\times12}{[G]} \tag{2.33}$$

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Element stiffness matrix

As usual stiffness matrix is given by

$$[K]=\int [B]^T\,[D]\,[B]\,dv \tag{2.34}$$

$$\underset{20\times20}{[K_L]} = \begin{bmatrix} \underset{8\times8}{K^L_{pp}} & \underset{8\times12}{0} \\ \underset{12\times8}{0} & \underset{12\times12}{K^L_{bb}} \end{bmatrix} \tag{2.35}$$

And

$$\underset{20\times20}{[K_N]} = \int_A \begin{bmatrix} 0 & \left[B^L_p\right]^t \left[D_p\right]\left[\bar{B}^N_p\right] \\ \left[\bar{B}^N_b\right]^t \left[D_p\right]\left[B^L_p\right] & \left[\bar{B}^N_b\right]^t \left[D_p\right]\left[\bar{B}^N_b\right] \end{bmatrix} dA \tag{2.36}$$

The equation is derived as

$$\int_A d\,\underset{20\times6}{\left[\bar{B}\right]}^t \underset{6\times1}{\{\sigma\}}\,dA = \int_A d\,\underset{20\times6}{\left[\bar{B}^N\right]}^t \underset{6\times1}{\{\sigma\}}\,dA$$

$$= \int_A \begin{bmatrix} 0 & d\left([A][G]\right) \\ 0 & 0 \end{bmatrix}^t \{\sigma\}\,dA$$

$$= \int_A \begin{Bmatrix} \underset{8\times1}{0} \\ [G]^t \underset{2\times2}{\begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix}} \underset{2\times12}{[G]}\underset{12\times1}{\{d\delta_b\}} \end{Bmatrix} dA$$

$$= \int_A \begin{bmatrix} 0 & 0 \\ 0 & [G]^t\,[T]\,[G] \end{bmatrix} \begin{Bmatrix} d\delta_p \\ d\delta_b \end{Bmatrix} dA \tag{2.37}$$

$[K_\sigma]$ is the matrix which accounts for the change in potential energy associated with rotation of volume elements under load. This matrix is called as 'initial stress stiffness matrix' or 'differential stiffness matrix' or geometric stiffness matrix or stability coefficient matrix. $[K_\sigma]$ is independent of elastic properties. It depends on elements geometry, displacement field and state of stress. The matrix can be expressed as:

$$\underset{20\times20}{[K\sigma]} = \int_A \begin{bmatrix} 0 & 0 \\ 0 & [G]^t\,[N]\,[G\,] \end{bmatrix} dA \tag{2.38}$$

$$\underset{12\times12}{\left[K^b_\sigma\right]} = \int_A [G]^t\,[N]\,[G]\,dA \tag{2.39}$$

Where,

$$\underset{2\times2}{[N]} = \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} \tag{2.40}$$

Total element stiffness matrix can be written as,

$$\underset{20\times20}{[K_T]} = \underset{20\times20}{[K_L]} + \underset{20\times20}{[K_N]} + \underset{20\times20}{[K_\sigma]} \tag{2.41}$$

$$\underset{20\times20}{[K_T]} = \begin{bmatrix} K^l_{pp} & K^N_{pb} \\ K^N_{bp} & K^L_{bb} + K^N_{bb} + K^b_\sigma \end{bmatrix} \tag{2.42}$$

III. Numerical Results and Discussions

Numerical examples of composite plates having different features are solved by the proposed element and the results obtained are presented with the published results for necessary comparison. Example 1: An anti-symmetric, cross-ply ($0^0/90^0$) square laminate, with which dimensions of a and b, in x and y directions respectively, is considered as subjected to a sinusoidal varying mechanical load of the type [q = qo sin px sin qy] with clamped boundary condition. The material properties used are as follows:

$E_1/E_2 = 25$, $G_{12}/E_2 = 0.5$, $G_{12} = G_{13}$, $G_{23}/E_2 = 0.2$, $\gamma_{12} = \gamma_{13} = \gamma_{23} = 0.25$, $E_2 = E_3 = 1.0$ x 106, a = b= 30, h =3

Table 3.1 Comparison of central deflection

Load Psi	Central Deflection 'w' in. (Present)	Central Deflection 'w' in. (Chang et al)
30000	2.8013	2.5877
60000	3.7025	3.4471
90000	4.3289	4.0372
120000	4.7908	4.4968
150000	5.2287	4.8821
180000	5.6158	5.2166
210000	5.9386	5.5143
240000	6.2625	5.7836
270000	6.3329	6.0306
300000	6.6892	6.2593

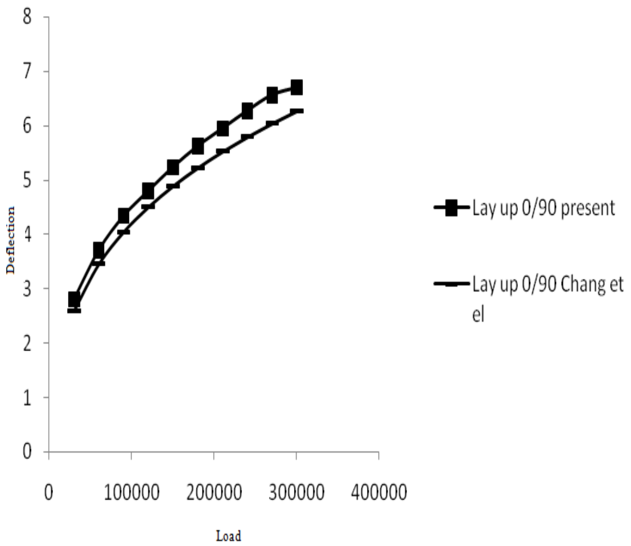

Fig. 3.1 Load – Deflection curve for cross-ply ($0^0/90^0$) square laminate,

Example 2: A 4 ply anti symmetric ($-45^0/45^0/45^0/-45^0$) square laminated plate with a = b = 12 in., h = 0.096 in.and subjected to a sinusoidal varying mechanical load of the type [q = qo sin px sin qy] is used with clamped edge conditions. The following material properties are used

$E_1 = 1.8282$ x 106, $E_2 = 1.8315$ x 106, $G_{12} = G_{23} = G_{13} = 3.125$ x 105, $\gamma = 0.23949$

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Table 3.2 Comparison of central deflection

Load psi	Central Deflection 'w' in. (Present)	Central Deflection 'w' in. (Chang et al)
30000	2.9581	3.0526
60000	3.9372	4.1088
90000	4.6389	4.8305
120000	5.1893	5.3913
150000	5.5718	5.861
180000	5.8773	6.2689
210000	6.2903	6.6317
240000	6.6402	6.9602
270000	6.9318	7.2614
300000	7.1991	7.5403

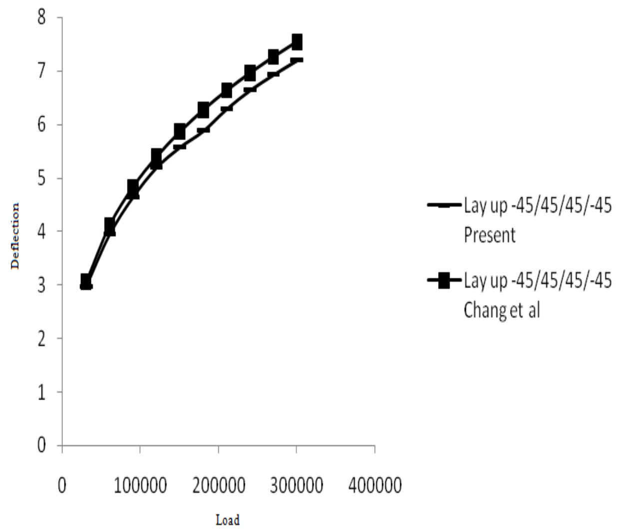

Fig. 3.2 Load – Deflection curve for anti symmetric (-450/450/450/-450) square laminated

Fig 3.1& Fig.3.2 show non-dimensional load versus non-dimensional maximum central deflection . The results obtained for central maximum deflection are compared with the results of Chang et al. (1991), and are found in good agreement. It is observed that maximum central deflection of laminated plate increases as load increases.

IV. CONCLUSION

A new rectangular element based on first order shear deformation theory is presented. The formulation is validated by comparing results with the relevant literature. The element is tested numerically in a wide range of problems covering different boundary conditions, loading, material property, and stacking sequence and so on. This study has lead to the following conclusions. The consideration of non-linear terms in the formulation results in better agreement of the response with both, the experimental and the analytical, solution in relevant literature. The central deflection increases with increase in the value of load. The central deflection decreases with increase in number of layers for the same thickness. It is observed that the central deflection reduces with increase in the degree of orthotropy and that the rate of change of transverse deflection with respect to degree of orthotropy is almost identical for both symmetric and anti-symmetric plies considered for the present study. From the present study it may be conclude that the above results will be helpful to structural engineers as design charts.

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