Modelling of ECG Signal using Fourier Series

Pooja Sabherwal

Abstract*— *In this paper ECG signal is generated from the concept of fourier series. It is based on the glimpse of generating ECG signal equations of cardiovascular disorder systems. The main idea is to develop the simulator for advanced applications. The ECG signal for normal person and diseased person is present. This simulator generates ECG signal for various cardiovascular diseases. These signals can be used for designing the computer based algorithms for automated diagnosis of the heart diseases. Here very simple method for generation of ECG signal based on fourier series is presented.

Index Terms*—*ECG Signal, Fourier Series,

I. Introduction

The electrocardiogram, ECG, provides useful information about functioning of heart required for cardiovascular assessment. ECG signal is a periodical wave measured on the surface of a human by ECG monitor or recorder. The sinoatrial node (SA node) is the pacemaker where the electrical impulse is generated. This node is located along the posterior wall of the right atrium right beneath the opening of the superior vena cava. It is crescent shaped and about 3 mm wide and 1 cm long. The impulse travels from the SA node through the internodal pathways to the atrioventricular node (AV node). The AV node is responsible for conduction of the impulse from the atria to the ventricles. The impulse is delayed slightly at this point to allow complete emptying of the atria before the ventricles contract. The impulse continues through the AV bundle and down the left and right bundle branches of the Purkinje fibers. The Purkinje fibers conduct the impulse to all parts of the ventricles, causing contraction (Guyton, 1982)[2].

In conventional 12 lead ECG systems, the electrodes at left arm, right arm& left leg are connected to a node through equal resistances. This node is called Wilson central terminal(WCT)[2]. Tilg et al[3] have developed Model which is based on bi domain theory of surface heart model. D. Phayung et al[4] have proposed a discrete mathematical model of ECG wave using kernel function. Three coupled ordinary differential equations are used to develop dynamical model of ECG which can generate realistic synthetic ECG signals is proposed by L.Sornmo et al [7]Patrick et al[6]. A discrete mathematical model of ECG wave has been proposed by S.Yimman et al[5] to develop an ECG simulator which can simulate ECG signal.

Here we propose the simulated model of ECG signal through the concept of Fourier series.

II. The ECG Signal

The ECG signal is a recording of body surface potentials generated by the electrical activity of the heart. It is a record of the direction and magnitude of electrical commotion that is generated by the heart muscles on depolarization and repolarization of the atria and ventricles. It is used for diagnosis of various cardiac abnormalities like identifying the cause of chest pain, in treating myocardial infarction, locating the cause of breathlessness, etc. One cardiac cycle in the ECG signal consist of P-QRS-T waves [1]. Fig. 1 shows a sample ECG signal.

The P wave is in congruous with contraction of the atria, and is roughly 0.2 mV in amplitude. The QRS complex concurs with depolarization of the ventricles, and has amplitude of approximately 1 mV. The T wave is associated with the return of ventricular mass to its resting electrical state (repolarization) and has amplitude of 0.1–0.3 mV in amplitude and ends 300–400ms after the beginning of the QRS complex. Algorithms for ECG beats detection focus on the QRS complex because its short duration and high amplitude make it the most prominent feature.

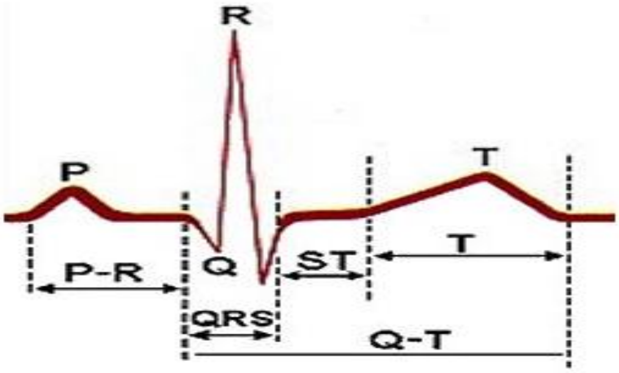

Fig.1 The ECG Signal

III. Principle of Fourier Series

The Fourier series is a specialized tool that allows for any periodic signal to be decomposed into an infinite sum of sinusoids. Any periodic functions which satisfy dirichlet's condition can be expressed as a series of scaled magnitudes of sin and cos terms of frequencies which occur as a multiple of fundamental frequency.

If we have a function $f(x)$, that is periodic with a period of 2l, we can decompose it into a sum of sine and cosine functions as such:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{--------------------------------(1)}$$

The coefficients, a and b can be found using the following integrals:

$$a_0 = \frac{1}{l}\int_T f(x)\,dx \qquad ,T = 2l \quad \text{--(2)}$$

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$a_n = \frac{1}{l}\int_T f(x)\cos\left(\frac{n\pi x}{l}\right)dx \qquad n = 1,2,3 \ldots\ldots\ldots$ ----------------------------------(3)

$b_n = \frac{1}{l}\int_T f(x)\sin\left(\frac{n\pi x}{l}\right)dx \qquad n = 1,2,3 \ldots\ldots\ldots$ ----------------------------------(4)

"n" is an integer variable. It can assume positive integer numbers (1, 2, 3, etc...). Each value of n corresponds to values for A and B. The sinusoids with magnitudes A and B are called harmonics.

When we set n = 1, the resulting sinusoidal frequency value from the above equations is known as the fundamental frequency. The fundamental frequency of a given signal is the most powerful sinusoidal component of a signal, and is the most important to transmit faithfully. Since n takes on integer values, all other frequency components of the signal are integer multiples of the fundamental frequency.

IV. METHDOLOGY

ECG signal is periodic with fundamental frequency determined by the heart beat. It also satisfies the dirichlet's conditions as the ECg signal is

• Single valued and finite in the given interval
• Absolutely integrable
• Finite number of maxima and minima between finite intervals
• It has finite number of discontinuities

A single period of a ECG signal is assumed as a mixture of triangular and sinusoidal wave forms. Other significant feature of ECG signal are represented by shifted and scaled versions of these waveforms as shown in fig 2 and fig 3.

• In a QRS complex, Q and S portions of the ECG signal are represented by triangular waveforms
• P, T and U waves are represented by sinusoidal waveforms

After the generation of QRS Complex, P ,T and U wave ,these all waves and complexes are finally added to get the ECG signal. The fig.2.shows the QRS complex which is generated through triangular waveforms.

QRS waveform is taken as the center point and all the shiftings takes place with respect to this part of the signal.

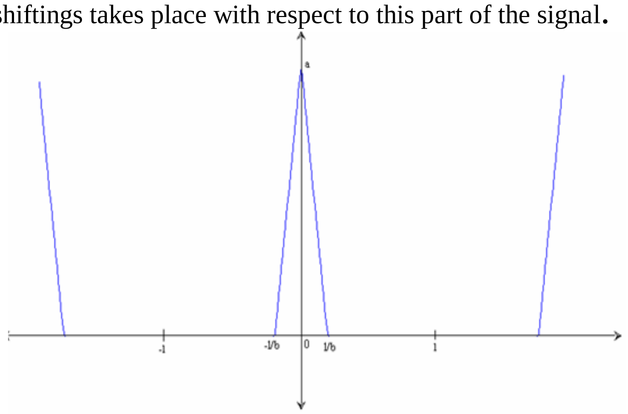

Fig.2. Generation of QRS complex

From (1) we have the equation of QRS complex as

$f(x) = \left(-\frac{bax}{l}\right) + a \qquad 0 < x < \frac{1}{b}$ ---(5)

$= \left(\frac{bax}{l}\right) + a \qquad -\left(\frac{1}{b}\right) < x < 0$

Where the integrals a and b are a given in (6),(7) and (8)

$a_0 = \frac{1}{l}\int_T f(x)\,dx = \frac{a}{b} * (2 - b)$ --(6)

$a_n = \frac{1}{l}\int_T f(x)\cos\left(\frac{n\pi x}{l}\right)dx = \left(\frac{2ba}{n^2\pi^2}\right) * \left(1 - \cos\left(\frac{n\pi}{b}\right)\right)$ --------------------------------------(7)

$b_n = \frac{1}{l}\int_T f(x)\sin\left(\frac{n\pi x}{l}\right)dx = 0$ *(because the waveform is a even function)* -----(8)

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$ -- (9)

Generation of P ,T and U wave

From (1) we have the equation of P,T and U wave as

$f(x) = \cos\left(\frac{\pi b x}{2l}\right) \quad \left(-\frac{1}{b}\right) < x < \left(\frac{1}{b}\right)$ ---(10)

Where the integrals a and b are a given in (11),(12) and (13)

$a_0 = \frac{1}{l}\int_T \cos\left(\frac{\pi b x}{2l}\right)dx = \frac{a}{2b}(2 - b)$ --(11)

$a_n = \frac{1}{l}\int_T \cos\left(\frac{\pi b x}{2l}\right)\cos\left(\frac{n\pi x}{l}\right)dx = \left(\frac{2ba}{l^2\pi^2}\right)\left(1 - \cos\left(\frac{n\pi}{b}\right)\right)\cos\frac{n\pi x}{l}$ -------(12)

$b_n = \frac{1}{l}\int_T \cos\left(\frac{\pi b x}{2l}\right)\sin\left(\frac{n\pi x}{l}\right)dx = 0$ *(because the waveform is a even function)* (13)

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$ ---(14)

The generated P wave is shown in Fig.3.

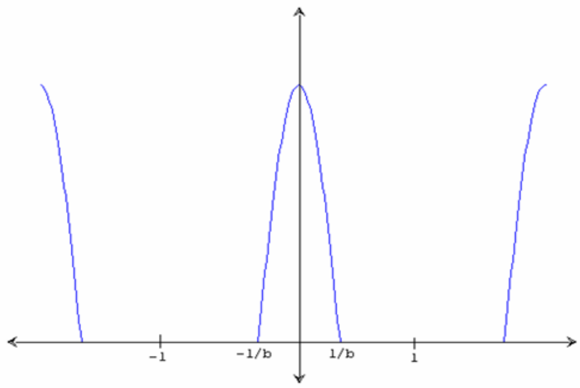

Fig.3. Generation of P wave

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V. RESULTS

The complete ECG signal is simulated on matlab. This ECG simulator has following features

- Any value of heart beat can be set
- Any value of intervals between the peaks (ex-PR interval) can be set
- Any value of amplitude can be set for each of the peaks
- Fibrillation can be simulated
- Noise due to the electrodes can be simulated
- Heart pulse of the particular ECG wave form can be represented in a separate graph

This simulator can be used for generation of normal and diseased signal. The below fig4 shows the simulated ECG signal for normal person and fig.5 shows the simulated ECG signal for diseased person.

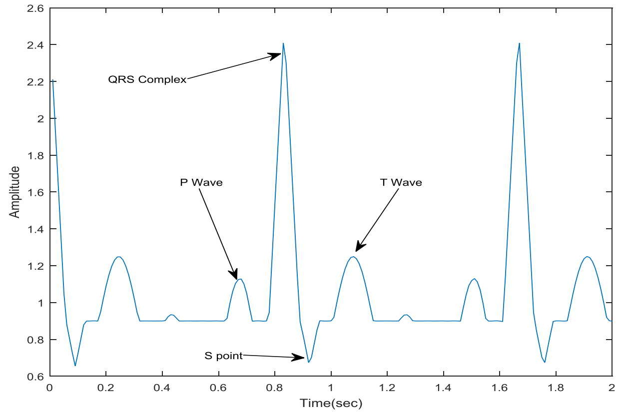

Fig.4-. ECG signal for normal person

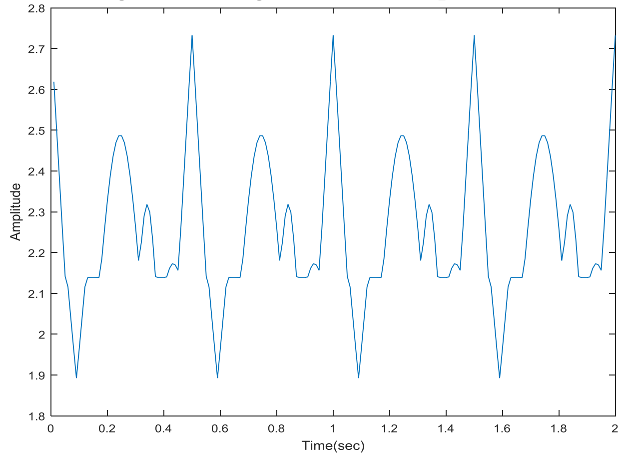

Fig.5-. ECG signal for diseased person

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