International Journal of Emerging Science and Engineering (IJESE)
ISSN: 2319–6378 (Online), Volume-6 Issue-6, October 2019

Mathematical Modeling of Oxygen Transport in Retinal Layers

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Abstract: ***The retina contains maximum oxygen (O_2) consumption making it particular valuneric to vascular infults,diminishing oxygen and circulation of nutrients.The goel of this paper to develop a mathematical model for quantitative analysis of transportation of oxygen and consumption in each retinal layer.Such models allows us to predict the effect of qualitatative change in retinal vasculature on oxygen supply in retinal layers.A differential equation , which used in this paper describing how the oxygen transport and cosumpt by the retinal layers.***

Keywords: Oxygen transport, Retinal capillaries, Oxygen metabolism, Oxygen consumption, , Retinal vascular occlusion, Intraretinal oxygen pressure.

I. INTRODUCTION

Oxygen is very necessary for retinal activities, as in another tissue, oxygen diffuses through the tissue passively from circulation. The demand of oxygen is very high, it can not preserve in the retina of other tissues, the circulation of oxygen in retinal layers must be continuous. The parameter of the model specially consumption of oxygen can be determined by fit this model to experimentally measured of oxygen tension is a function of distance from first retinal layer.

Revised Manuscript Received on October 22, 2019.
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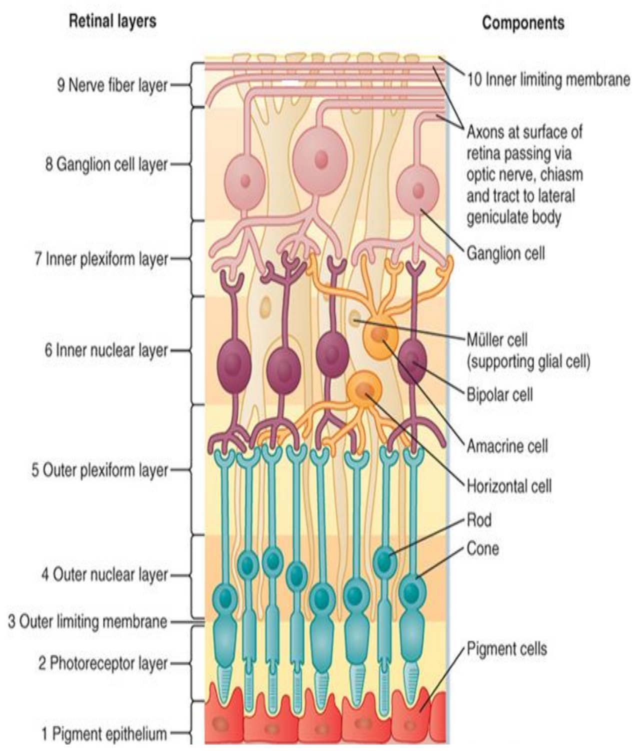

Fig:1

Retina is very congested and structured tissue which covers the inner area of the behind of the eye. A multilayered mathematical model described transport of oxygen and consumption by the human retina . The model takes the profit of highly layered structured of retina and available oxygen sources compartmentally. The retina divided into ten layers, each layer has distinct oxygen consumption and rate of oxygen transport depend on diffusion coefficient of concern layers[1,2]. This model will use to investigate the oxygen regulatory mechanism in the human retina during systematic hyperoxia. The circulation of oxygen to retinal layers is very crucial function .The delivery of oxygen to retina is a combination of the coroidal vascular, which situated back of retina and many diffusion reaction models are proposed for oxygen transport in retina. The blood is fully responsible to transport oxygen in the retinal layers and vessels walls are responsible to filter into the surrounding of tissue, where the quantity of perfuse solute depends on the tissue metabolic requirement. The model is valid against the available experimental result.

Retina required maximum metabolic demand for oxygen that is mached by highly efficient vascular circulation. Oxygen plays a very important role in oxidative as an electron receiver in mitochondrial respiratory system in the synthesis of adenosine tri phosphate required help to metabolic demand. There are two types of vascular supply to retina, given by independent and physiologically circulation. The deep vascular plexi generated from middle retinal artery also inner retina of human protected by superficial and outer retina is supplied by choroidal circulation deep to retina pigment epithelium and its basement membrane [8] .The effect of oxygen tension across retina has been evaluated by using polar graphic electrode is several species including pig [9] ,cats[4,7] rats [10].

In the present model we see that the consumption of oxygen increase first layer to last layer. Flow of blood and delivery of oxygen within a retinal tissue is an area of this research activity. It is very difficult to simulate computationally due to the complex structure of blood vessels and the range of sizes of vessels involved. Our purpose here to generate a mathematical and computational model shows the bonding between flow of blood activity and transport of oxygen in the retinal layers.

The sum of thickness of Inner retinal layers IPL+NFL+INL+GCL+ RPE +OPL had to be calculated (170 ±39) microns and consistent with measurements (160 ± 32 microns) to compare results with reported techniques that measure thickness of up to 5 layers. The sum thickness of outer retinal Layers +ONL+OPL+PR+RPE+ELM thickness measurements reported previously were 78 ± 10 microns, similar to thickness measurements of 77 ± 11microns derived for PIS+ONL by. Inner retina complex (IRC) were reported to be 28.4 ± 4.7 and 90.7 ± 4.2 microns and thickness measurements of NFL.2 These values are optimal with thickness measurements of IPL+ GCL+ INL (95± 17 microns) and NFL (30 ± 6 microns) [11].

review process. All submitted manuscripts are reviewed by three reviewer one from India and rest two from overseas. There should be proper comments of the reviewers for the purpose of acceptance/ rejection. There should be minimum 01 to 02 week time window for it.

Governing equations:

The steady state oxygen consumption equation is governed by

$$D_i \frac{d^2 q_i}{dx^2} - q_i + s_i = 0 \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

Where D_i=Diffusion coefficient of oxygen in the i^{th} retinal layer

q_i=The oxygen consumption in the i^{th} layer

s_i = Oxygen delivery in the i^{th} layer.

Solution of equation (1) is

$$q_i = c_1 \cosh \frac{1}{\sqrt{D_i}} x + c_2 \sinh \frac{1}{\sqrt{D_i}} x + s_i \dots\dots\dots\dots\dots\dots (2)$$

Where c_1 & c_2 depend on the value of Diffusion Coefficient and Depth of Retinal Layer.

Also by Michaelis-Menten Kinetics we have

$$q_i = \frac{p_i q_{(\max)i}}{p_i + \sigma_i} \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

p_i=Partial pressure of oxygen in the i^{th} layer[1,5,6]

$q_{(\max)i}$= Maximal consumption in the i^{th} layer

σ_i= Partial pressure of oxygen at half maxima

$$s_i = \frac{b_f}{60}\left[p^b - \gamma p_i + \frac{(p^b)^n}{(p^b)^n + (khem)^n} - \frac{\gamma p_i}{(\gamma p^b)^n + (khem)^n}\right]\frac{Hb.\varphi}{\delta} \dots\dots\dots\dots (4)$$

[1]

Where

b_f=blood flow rate in inner retinal layer[3]

γ=Constant

p^b = Partial pressure in arterial blood

Hb=The hemoglobin concentration in blood.

φ=The oxygen carrying capacity of hemoglobin

δ=Solubility of oxygen in blood.

n =Hill coefficient

$k\,hem$ = A constant which equal to the partial pressure of oxygen at which hemoglobin is 50% saturated with oxygen.

Using equation (4) , (3) become

$$q_i = c_1 \cosh \frac{1}{\sqrt{D_i}} x + c_2 \sinh \frac{1}{\sqrt{D_i}} x + \frac{b_f}{60}\left[p^b - \gamma p_i + \frac{(p^b)^n}{(p^b)^n + (khem)^n} - \frac{\gamma p_i}{(\gamma p^b)^n + (khem)^n}\right]\frac{Hb.\varphi}{\delta} (5)$$

Where $i = 1,2,3 \dots\dots\dots .10$

The value of c_1 & c_2 will be different layer wise

We will find the value of c_1 & c_2 by using following boundary condition

Outer half retinal layers has same oxygen consumption rate[2] and half inner retinal layers has same oxygen consumption.

$$(q_1)_{x=0} = 0 \dots\dots\dots\dots (6)$$

$$(q_1)_{x=L_1} = q_{inner\ limiting\ membrane} \dots\dots\dots\dots (7)$$

$$\left(\frac{\partial q_1}{\partial x}\right)_{x=L_1} = 2.86 \dots\dots\dots\dots (8)$$

$$(q_2)_{x=L_2} = q_{nerve\ fiber\ layer} \dots\dots\dots\dots (9)$$

$$\left(\frac{\partial q_2}{\partial x}\right)_{x=L_2} = 3.18 \dots\dots\dots\dots (10)$$

$$(q_3)_{x=L_3} = q_{ganglion\ cell\ layer} \dots\dots\dots\dots (11)$$

$$\left(\frac{\partial q_3}{\partial x}\right)_{x=L_3} = 3.5 \dots\dots\dots\dots (12)$$

$$(q_4)_{x=L_4} = q_{inner\ plexiform\ layer} \dots\dots\dots\dots (13)$$

$$\left(\frac{\partial q_4}{\partial x}\right)_{x=L_4} = 3.82 \dots\dots\dots\dots (14)$$

$$(q_5)_{x=L_5} = q_{inner\ nuclear\ layer} \dots\dots\dots\dots (15)$$

$$\left(\frac{\partial q_5}{\partial x}\right)_{x=L_5} = 4.14 \dots\dots\dots\dots (16)$$

$$(q_6)_{x=L_6} = q_{outer\ plexiform\ layer} \dots\dots\dots\dots (17)$$

$$\left(\frac{\partial q_6}{\partial x}\right)_{x=L_6} = 2.78 \dots\dots\dots\dots (18)$$

$$(q_7)_{x=L_7} = q_{outer\ nuclear\ layer} \dots\dots\dots\dots (19)$$

$$\left(\frac{\partial q_7}{\partial x}\right)_{x=L_7} = 3.34 \dots\dots\dots\dots (20)$$

Retrieval Number: F2304086619/2019©BEIESP
DOI: 10.35940/ijese.F2304.106619
Journal Website: www.ijese.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

International Journal of Emerging Science and Engineering (IJESE)
ISSN: 2319–6378 (Online), Volume-6 Issue-6, October 2019

$$(q_8)_{x=L_8} = q_{outer\ limiting\ layer} \dots\dots (21)$$

$$\left(\frac{\partial q_8}{\partial x}\right)_{x=L_8} = 3.9 \dots\dots (22)$$

$$(q_9)_{x=L_9} = q_{photo\ receptor\ layer} \dots\dots (23)$$

$$\left(\frac{\partial q_9}{\partial x}\right)_{x=L_9} = 4.46 \dots\dots (24)$$

$$(q_{10})_{x=L_{10}} = q_{retinal\ pigmented\ epithilium} \dots\dots (25)$$

$$\left(\frac{\partial q_{10}}{\partial x}\right)_{x=L_{10}} = 5.02 \dots\dots (26)$$

Outer half retinal layer has the rate of oxygen consumption $3.9 \pm 2.8\ \frac{ml\,O_2}{100gmin}$ and inner retinal layers has the rate of oxygen consumption $3.5 \pm 1.7\ \frac{ml\,O_2}{100gmin}$ with respect to time[7].

Maximal oxygen consumption rate in the light is $90mm/Hg$ and is $170mm/Hg$ in dark with respect to partial pressure in outer retina[12]. Also the maximal oxygen consumption rate for inner retinal layers is $26\ mm/Hg$ for dark as well as light [12,1].

Table:1[1]

Symbol	Explanation	Numerical Value	Unit
$D_i\ (i = 1,2,3 \dots 10)$	Diffusion Coefficient of Oxygen	2×10^{-5}	$\frac{cm^2}{s}$
$q_{(max)i}$	Maximal consumption in the i[th] layer	Layer3:90 in light 170 in dark Layer8:27 in light as well as dark	mmHg -1
σ_i	The partial pressure of oxygen at half maximal speed in the i^{th} layer	2	MmHg
p_i	The partial pressure of oxygen in the choriocapillaries.	80 (normoxia) 250 (hyperoxia) 405 (extrme - hyperoxia)	mm Hg
k_{hem}	A constant which equals the partial pressure of oxygen at which hemoglobin is 50% saturated with oxygen	26	MmHg
n	Hill Coefficient	2.7	
p^b	Partial pressure of oxygen in arterial blood	80,250,405	MmHg
Hb	The hemoglobin concentration in blood	140	$\frac{g}{L}$
φ	The oxygen carrying capacity of hemoglobin	.0616	$\frac{mmol}{g}$
b_f	blood flow rate in inner retinal layer	.4,.3,.2,.1,0	$\frac{ml}{g}m$
δ	Solubility of oxygen in blood	1.5×10^{-3}	$mMmmHg^{-1}$

Tables-

Sr.no	Outer Retinal Layers(Consumption rate wrt to depth) cm2
1	2.78
2	3.34
3	3.9
4	4.46
5	5.02
Sr.No	Inner retinal layers cm2
1	2.86
2	3.18
3	3.5
4	3.82
5	4.14

Figures

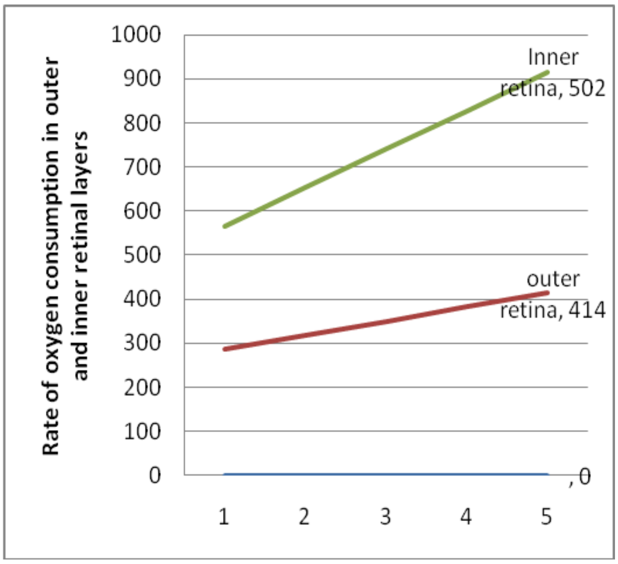

Fig:2

Comparison Graph between rate of oxygen consumption inner retinal layers and outer layers

From Above diagram we see that the consumption rate of oxygen in outer retinal layers is less than consumption rate of oxygen to inner retinal layers so if oxygen enters inside the retinal layers then the consumption of oxygen

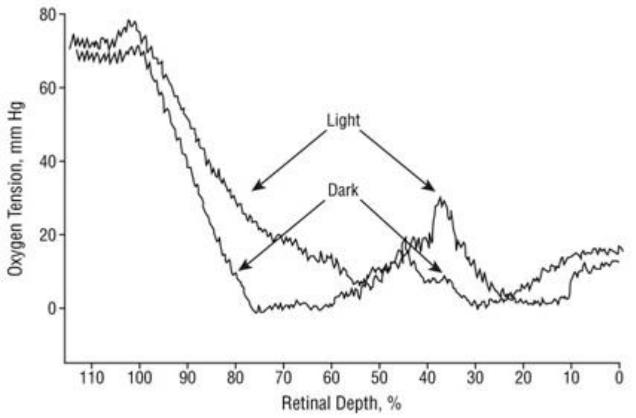

Fig:3

Retinal oxygen cosumption across retina during light and dark environment. The retina is shown schematically at the top. The rod photoreceptors, retinal pigment epithelial cells, ganglion cells and , bipolar cells are shown as left to right increase as well as depth of retina increase . The innermost layer consume maximum oxygen.

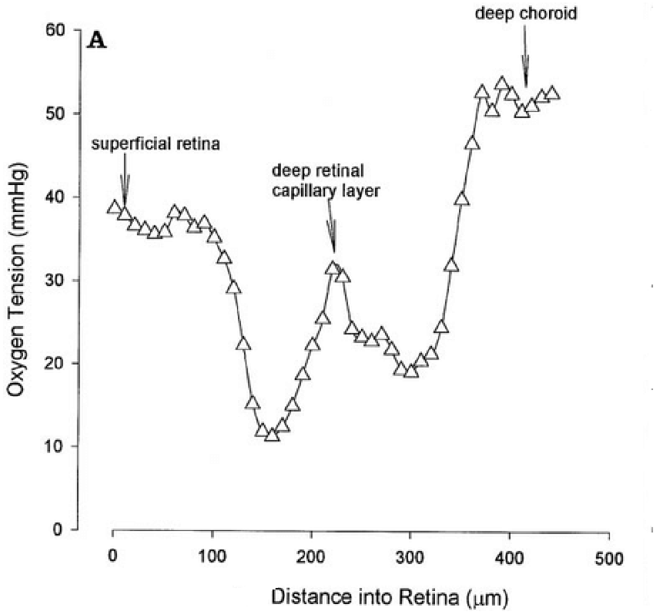

Fig:4

The oxygen tension(PO_2) is the function of distance from the first layer of retina.

II. RESULT AND DISCUSSION

This Model can use to give best treatment of Retina. As above discussion Oxygen is very important requirement of eye. By experimentally identify the affected layer and give full treatment to affected retinal layer, which will be less expensive as well as less time consuming and we will get better results rather than full eye treatment.

III. CONCLUSION

Low supply of oxygen to the retinal layers is root cause of retinal problems such as Retinal detachment ,retinal pigmentosa etc. Maximum circulation of oxygen in retinal layers is also cause of disease like hyperoxia. It has also proved that impure oxygen play a main role to regenerate the retinal diseases.

APPENDIX

.ACKNOWLEDGMENT

The authors gratefully acknowledge the constructive and fruitful comments of the reviewers of original manuscript of the paper.

REFERENCES

1. RamAvtar;Deepti Tandon ; 'Mathematical Modelling of Intraretinal Oxygen Partial Pressure" Tropical Journal of Pharmaceutical Research, December 2008; 7 (4): 1107-1116.
2. SJ Cringle;etal 'Intraretinal oxygen consumption in the rat in vivo' Invest. Ophthalmol Vis Sci. 2002; 43: 1922- 1927.
3. WM Roos. 'Theoretical estimation of retinal oxygenation during retinal artery occlusion'. Physiol. Meas. 2004; 25 :1523-1532.
4. VA Alder; SJ Cringle; 'PO2 profiles and oxygen consumption in cat retina with an occulded retinal circulation' . Invest. Ophthalmol. Vis.Sci. 1990;31:1029-1034.
5. Ram Avtar ; Deepti Seth ; Aadesh Kumar ; 'Mathematical Model of Oxygen Transport in Cornea' ;World Scientific News,An International Scientific Journal 2018,113(169-184).
6. Causin;etal 'Blood flow mechanics and oxygen transport and delivery in the retinal microcirculation: multiscale mathematical modeling and numerical simulation. Biomechanics & Modeling in Mechanobiology' , June 2016, Vol. 15 Issue 3, p525-542.
7. D Rod Braun ;etal 'Oxygen Consumption in the Inner and Outer retina of the cat' ;1995,36:542-554
8. JM Provis ; ' Development of the primate retinal vasculature'. Prog Retin Eye Res 2001; 20:799–821.
9. Pournaras CJ, Riva C; 'Tsacopoulos M,Strommer K: Diffusion of O 2 in the retina of anesthetized miniature pigs in normoxia and hyperoxia'. Exp Eye Res 1989; 49: 347–360.
10. DY Yu; etal ' Intraretinal oxygen distribution in rats as a function of systemic blood pressure'. Am J Physiol 1994; 267:H2498–H2507.
11. Rashid Ansari; Michael Blair etal 'Thickness Profiles of Retinal Layers by Optical Coherence Tomography Image Segmentation'*Am J Ophthalmol* ,2008 November ; 146(5): 679–687.
12. M. D Abramoff,., etal ,. 'Human photoreceptor outer segments shorten during light adaptation. Invest'. Ophthalmol. 2009, 3721–3728. 1581.

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